

Dokazovanje veljavnosti argumentov z metodo naravne dedukcije III

Sekundarna pravila			
(MT)	$P \supset Q$ $\neg Q$ $\therefore \neg P$	(EFQ)	$P, \neg P$ $\therefore Q$
(TZ)	$P \supset Q, Q \supset R$ $\therefore P \supset R$	(MI)	$P \supset Q, \therefore \neg P \vee Q$ $P \supset Q, \therefore \neg(P \wedge \neg Q)$ $\neg P \vee Q, \therefore P \supset Q$ $\neg(P \wedge \neg Q), \therefore P \supset Q$
(ABS)	$P \supset Q$ $\therefore P \supset (P \wedge Q)$	(KM)	$P \wedge Q, \therefore Q \wedge P$ $P \vee Q, \therefore Q \vee P$
(V IZ \supset)	$P \vee Q$ $P \supset R, Q \supset R$ $\therefore R$	(AS)	$(P \wedge Q) \wedge R, \therefore P \wedge (Q \wedge R)$ $P \wedge (Q \wedge R), \therefore (P \wedge Q) \wedge R$ $(P \vee Q) \vee R, \therefore P \vee (Q \vee R)$ $P \vee (Q \vee R), \therefore (P \vee Q) \vee R$
(KD)	$P \vee Q$ $P \supset R, Q \supset S$ $\therefore R \vee S$	(DB)	$(P \wedge Q) \vee R, \therefore (P \vee R) \wedge (Q \vee R)$ $(P \vee R) \wedge (Q \vee R), \therefore (P \wedge Q) \vee R$ $(P \vee Q) \wedge R, \therefore (P \wedge R) \vee (Q \wedge R)$ $(P \wedge R) \vee (Q \wedge R), \therefore (P \vee Q) \wedge R$
(KP)	$P \supset Q$ $\therefore \neg Q \supset \neg P$		
(DM)	$\neg P \vee \neg Q, \therefore \neg(P \wedge Q)$ $P \vee Q, \therefore \neg(\neg P \wedge \neg Q)$ $\neg(P \wedge Q), \therefore \neg P \vee \neg Q$ $\neg(\neg P \wedge \neg Q), \therefore (P \vee Q)$	$\neg P \wedge \neg Q, \therefore \neg(P \vee Q)$ $P \wedge Q, \therefore \neg(\neg P \vee \neg Q)$ $\neg(P \vee Q), \therefore \neg P \vee \neg Q$ $\neg(\neg P \vee \neg Q), \therefore P \wedge Q$	

Vaja: Z naravno dedukcijo dokaži veljavnost argumenta.

<1> $\neg(p \vee q), \therefore \neg p \wedge \neg q$

rešitev:

1. $\neg(p \vee q)$	d		1. $\neg(p \vee q)$	d
2. p	h (za RA)		2. $\neg p \wedge \neg q$	(DM)
3. p \vee q	(VV):(2)			
4. (p \vee q) \wedge \neg (p \vee q)	(\wedge V):(1),(3)			
5. \neg p	(RA):(2)-(4)			
6. q	h (za RA)			
7. p \vee q	(VV):(6)			
8. (p \vee q) \wedge \neg (p \vee q)	(\wedge V):(1),(7)			
9. \neg q	(RA):(6)-(8)			
10. \neg p \wedge \neg q	(\wedge V):(5),(9)			

<2> $\neg p \supset q, r \supset s, \neg p \vee r, \neg q, \therefore s$

rešitev:

1. $\neg p \supset q$	d			
2. $r \supset s$	d			
3. $\neg p \vee r$	d			
4. $\neg q$	d			
5. $\neg\neg p$	(MT):(1),(4)		5. $q \vee s$	(KD):(1),(2),(3)
6. p	($\neg\neg$ IZ):(5)		6. s	(VIZ):(4),(5)
7. r	(VIZ):(3),(6)			
8. s	(MP):(2),(7)			

<3> $p \supset q, (p \wedge q) \supset r, \neg r, \therefore \neg p$

<4> $(\neg p \wedge q) \supset \neg r, r, q, \therefore p$

<5> $p \supset (q \supset \neg r), r, \therefore \neg p \vee \neg q$

<6> $\neg p \vee \neg q, r \supset p, \neg\neg q \vee \neg s, \therefore \neg s \vee \neg r$

<7> $p \equiv q, \therefore \neg((p \supset q) \supset \neg(q \supset p))$

<8> $(p \supset q) \wedge (p \supset r), \therefore p \supset (q \wedge r)$

$$\langle 9 \rangle p \wedge q, \therefore p \supset q$$

$$\langle 10 \rangle (\neg p \wedge q) \supset \neg r, r, q, \therefore p$$

$$\langle 11 \rangle p \equiv (q \wedge r), \neg \neg q, \therefore r \supset p$$

$$\langle 12 \rangle (p \vee \neg q) \supset (\neg r \wedge s), (\neg r \vee s) \supset t, p, \therefore t$$

$$\langle 13 \rangle (\neg \neg p \supset r), (q \supset r), \therefore (p \vee q) \supset r$$

$$\langle 14 \rangle p \wedge q, p \supset (r \vee s), q \wedge \neg r, \therefore (s \wedge p) \wedge q$$

$$\langle 15 \rangle p \supset (q \vee (r \wedge s)), p \wedge \neg q, \therefore s$$

$$\langle 16 \rangle (p \wedge q) \supset \neg r, p \wedge r, \therefore \neg q$$

$$\langle 17 \rangle p \equiv (\neg q \wedge r), \neg r, \therefore \neg p$$

$$\langle 18 \rangle p, (p \wedge q) \supset \neg r, \neg r \supset \neg s, \therefore q \supset \neg s$$