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Paradoxes of Transfinite Cosmology

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Abstract of the paper version

After a short survey of the main historical concepts of infinity, especially in Aristotel, Kant and Cantor, the question is considered whether the modern cosmology has solved the first Kant's antinomy, i.e., whether the universe is finite or infinite in space and time, and in this context also some topological issues are discussed (torus etc.). Then the concept of “multiverse” is analyzed, and some recent cosmological theories of multiverse(s) are discussed from the methodological point of the set-theory. If multiverses are conceived as sets of universes, then the concept of “Multiverse of all multiverses” yields to be paradoxical, and it seems that a return to some “meta-concept” of the Universe is unavoidable. In this sense, Cantor’s “Absolute” is proposed as a possible philosophical solution of the paradoxes of infinity also for cosmological multiverses, and in the conclusion of the paper, the analogy between Cantor’s Absolute, which is not a “mathematical object”, and Kant’s conception of the Whole of the World (in space and time etc.), which is not a constitutive category of reality, but just a “regulative idea” of transcendental dialectical thought, is proposed and discussed.

Key words: infinity, cosmology, multiverse, paradox, antinomy, Kant, Cantor.

The “Multiverse Hypothesis”

There are two main motives for introducing the “Multiverse Hypothesis” (or the “Many-Worlds Hypothesis”), i.e., to postulate the existence of many worlds/universes in physical (meta)theories:

1) the problem of interpretation of quantum states,
2) the problem of the cosmological “fine tuning” of basic physical constants (“free parameters”).

“Fine-tuning” is either real or apparent.

- In the first case, it should be the consequence of some kind of teleology of nature (transcendent or immanent),
- in the second case it could be, in principle, explained in two ways:
    - ➢ either by some future “Final Theory” or “Theory of Everything” (?)
    - ➢ or by the “Observation Selection Effect” which requires the “Multiverse Hypothesis”.

In cosmology, the latter option is also known as the “anthropic reasoning” which has been initiated by Brandon Carter (1974) with his “Anthropic Cosmological Principle”.

The “Anthropic Cosmological Principle”

Brandon Carter’s “Anthropic Cosmological Principle” (1974) has in its original formulation two versions:

- *Weak Anthropic Principle*: “We must be prepared to take account of the fact that our location in the universe is *necessarily* privileged to the extent of being compatible with our existence as observers.”
- *Strong Anthropic Principle*: “The Universe (and hence the fundamental parameters on which it depends) must be such as to admit the creation of observers within it at some stage. To paraphrase Descartes, *Cogito ergo mundus talis est*.”

Hierarchy of multiverses

Max Tegmark is his paper "The Multiverse Hierarchy" (2003), which has been reprinted in *Universe or Multiverse?* (ed. Bernard Carr, 2007), classifies multiverses into four-level hierarchy, which progressively yields greater diversity of universes:

- "Level I: A generic prediction of cosmological inflation is an infinite 'ergodic' space, which contains Hubble volumes realizing all initial conditions – including one with an identical copy of you about 10 on 10^{29} power meters away.
- Level II: Given the *fundamental* laws of physics that physicists one day hope to capture with equations on a T-shirt, different regions of space can exhibit different *effective* laws of physics (physical constants, dimensionality, particle content, etc.), corresponding to different local minima in a landscape of possibilities.
- Level III: In unitary quantum mechanics, other branches of the wave-function add nothing qualitatively new, which is ironic given that this level has historically been the most controversial.
- Level IV: Other mathematical structures give different fundamental equations of physics for that T-shirt." (Tegmark, in Carr 2007: 99-100)

“Complete mathematical democracy”

Multiverses in Tegmark’s four-level hierarchy derive from:

I. different initial conditions

II. different “effective” physical laws

III. “parallel branches” of quantum states

IV. different “underlying” mathematical structures which give different fundamental physical laws.

Level-IV incorporates the idea of the “complete mathematical democracy”, which means that “a mathematical structure and the physical world are in some sense identical” (*ibid*. 116).

– otherwise said, that “each physical entity [has] a unique counterpart in the mathematical structure and vice versa” (*ibid*. 117).

However, it would also mean that there is no “free mathematics” (in Cantor’s sense) at all.

Simplicity of the “Multiverse Hypothesis”?

The *first* paradox of the multiverse cosmology is its putative *simplicity*.

Tegmark’s epistemological (in)version of Ockham’s razor is quite typical for proponents of multiverse(s):

- “A common feature of all four multiverse levels is that the simplest and arguably most elegant theory involves parallel universes by default. To deny the existence of those universes, one needs to complicate the theory by adding experimentally unsupported processes and ad hoc postulates: finite space, wave-function collapse, ontological asymmetry, etc. Our judgment therefore comes down to which we find more wasteful and inelegant: many worlds or many words. Perhaps we will gradually become used to the weird ways of our cosmos, and even find its strangeness to be part of its charm.” (Tegmark, in Carr 2007: 123-25).

The *second* and the main paradox (or a cluster of paradoxes) of the multiverse cosmology, a paradox in a strict sense, emerges from the question where is the *limit* of the ascending hierarchy of universes and/or multiverses – does they rise up to *infinity*? After all, infinity is much “simpler” in Tegmark’s sense than some specific “googol number” …

Two philosophical questions

The central part of my paper is a discussion about some paradoxical and philosophically interesting epistemo-logical aspects of the following two questions:

1) What is the relation between mathematical and physical infinity, especially in case if a general “structural isomorphism” between mathematics and physics obtains, as Tegmark claims.
2) Does the metaphysical and/or theological infinity *transcend* the physical and/or mathematical infinity?

Considering the concept of “transfinite cosmology”

I think that from Tegmark’s Level-IV of multiverse, which is conceived from the viewpoint of “mathematical democracy”, it is unavoidable to consider also the concept of “transfinite multiverses”, *per analogiam* with Cantor’s transfinite sets.

Roger Penrose in “The Ladder of Infinity”, the Chapter 16 of his great book *The Road to Reality*, wrote:

- “It is perhaps remarkable, in view of the relationship between mathematics and physics, that issues of such basic importance in mathematics as transfinite set theory and computability have as yet had a very limited impact on our description of the physical world. It is my own personal opinion that we shall find that computability issues will eventually be found to have a deep relevance to future physical theory, but only very little use of these ideas has so far been made in mathematical physics.” (Penrose 2004: 378)

Problems with definition of the set *M*

The role of mathematical sets in physical theories of multiverses is well considered in the article "Multiverses and Cosmology: Philosophical Issues" by authors **William Stoeger, George Ellis, Uli Kirchner** (2006):

As they point out, in any methodologically and conceptually well-formed theory of multiverses, it is necessary –

1) first to *define* a set *M*, whose elements are all *possible* universes *m*,
2) then to determine a "distribution function" *f*(*m*), that selects within *M* actually *existent* universes,
3) and finally a criterion (also a function) that determines the *anthropic* subset among existent universes.

– but we have a heavy problem already in how to define *M*:

- "What determines *M* ? Where does this structure come from? What is the meta-cause, or ground, that delimits this set of possibilities? Why is there a uniform structure across all universes *m* in *M* ?" (Stoeger & Ellis & Kirchner 2006: 7)

Mathematical and physical infinities

There are unsolvable problems with physical (“actual”) infinities:

- “When speaking of multiverses or ensembles of universes – possible or realized – the issue of infinity inevitably crops up. Researchers often envision an infinite set of universes, in which all possibilities are realized. Can there be an infinite set of really existing universes? We suggest that the answer may very well be ‘No’.” (Stoeger & Ellis & Kirchner 2006: 13)

The three authors refer to **David Hilbert**’s thought that “the presumed existence of the actually infinite directly or indirectly leads to well-recognized unsolvable contradictions in set theory” (*ibid*. 14).

- “[T]he problem with a realized infinity is not primarily physical in the usual sense – it is primarily a conceptual or philosophical problem. ‘Infinity’ as it is mathematically conceived […] really refers to a process rather than to an entity […] And the process it refers to has no term or completion specified. No physically meaningful parameter really possesses an infinite value.” (*Ibid*. 17)

Kant's cosmological antinomy of space and time

- "*Thesis*: The world has a beginning in time, and in space it is also enclosed in boundaries.
- *Antithesis*: The world has no beginning and no bounds in space, but is infinite with regard to both time and space."

(Immanuel Kant, *Critique of Pure Reason*, B454 and B455, tr. by Paul Guyer and Allen W. Wood, Canbridge U.P., 2007)

Cantor’s concept of “inconsistent wholes”

Cantor presumed that some sets (“wholes”) are so disproportionally large that it is impossible to assign any “power” (i.e., any cardinal number) to them, as it can be assigned to countable infinity or to continuum.

A. W. Moore stated in his book *The Infinite*: “There was [for Cantor] no such set as Ω [the “whole” of all ordinal numbers]. And this was enough to dispel the [Burali-Forti’s] paradox” (Moore 1990: 127).

Cantor named such concepts “inconsistent wholes”: they do not belong to the transfinite domain, but to the “domain” of *absolute* infinity, in short – to the Absolute.

Therefore, there is *no* “Cantor’s paradox of the greatest Aleph”.

Was Cantor in his conception of a set indeed "naive"?

Shaughan Lavine in his book *Understanding the Infinite* asserts that Cantor actually did *not* accept the "Comprehension Principle" (CP) which says that every property (or propositional function) determines a set.

However, it is not quite clear from the famous Cantor's definition of a set in the *Grundlagen* (1883) whether he was in this respect "naive" or not:

- "By 'set' [*'Mannigfaltigkeit' oder 'Menge'*] I understand in general every many that can be thought of as one, i.e., any domain of definite elements which by means of a law can be bound up into a whole..." (Cantor, quoted from Lavine 1994: 85).

Lavine thinks that Cantor conceived a set as a "combinatorial collection", which is "defined by the enumeration of its terms" (*ibid*. 77) – and thereby avoided "naivity" and paradoxes.

William W. Tait in his article "Cantor's *Grundlagen* and the Paradoxes of Set Theory" expresses a different view, saying that "with the introduction of transfinite numbers, Cantor immediately recognized that the notion of set was problematic" (Tait 2000: 23) – anyway, he also points out that Cantor distinguished between the "determinate infinities" and the "absolute infinite" (*ibid*. 11), and just *this* distinction is essential for our philosophical context.

Cantor’s letter to Jourdain on July 9, 1904

- “Were we now, as Mr. Russell proposes, to replace *M* by an *inconsistent* multiplicity (perhaps by the totality of *all* transfinite ordinal numbers, which you call *W*), then a totality corresponding to *G* could *by no means be formed*. The impossibility rests upon this: an inconsistent multiplicity because it cannot be understood as a *whole*, thus as a *thing*, can*not* be used as an *element* of a multiplicity.

  Only *complete things* can be taken as *elements* of multiplicity, only *sets*, but not *inconsistent multiplicities*, in whose nature it lies, that they can never be conceived as *complete* and *actually existing*.”

  (Georg Cantor, quoted from Lavine 1994: 99)

Cantor's transcendent conception of the Absolute

Cantor, in his early series of six treatises, titled *Ueber unendliche, lineare Punktmannichfaltigkeiten* ("About infinite, linear manifolds of points", 1879-84, the 5th is known as *Grundlagen*, 1883), wrote in the context of his critique of Aristotle's "potential infinity" the following, philosophically very significant thoughts (*Grundlagen*, § 4, note 2):

- "Plato's concept of infinity is quite different from Aristotle's […] I have found contact points for my conceptions also in the philosophy of Nicholas Cusanus. […] And I notice the same in Giordano Bruno, the follower of Cusanus. […] However, there is an essential distinction, namely that I have once for all fixed in concept the different degrees of the actual infinite with the classes of numbers (I), (II), (III) etc. […] I do not doubt that we will go on and on in this way, and that we will never encounter some impassable boundary, but that we shall also not succeed in approaching to some merely near comprehension of the Absolute. The Absolute can only be acknowledged, but never be known, it cannot be even nearly known." (Translated by M.U.)

Three levels of infinity, following Cantor

1) The “improper” (although in mathematics indispensable) infinity of addition and division, which Aristotle named “potential infinity”;
2) the “proper” (actual) infinity of transfinite numbers, ordinals and cardinals, which Cantor himself discovered; and
3) the transcendent infinity of the Absolute, which is only *symbolically* recognized in the mathematical infinity, but is never conceptually known. – “Therefore, the absolute infinite series of numbers seems to me in a certain sense as an adequate symbol of the Absolute” (Cantor, *ibid*. 116, tr. by M.U.)

Conclusions

1) Cantor’s conception of the absolute infinity is spiritually akin to Kant’s critical philosophy. For Kant, the cosmological “totality” is a *transcendent*, dialectical “regulative idea” of the pure reason <*Vernunft*>, but it is *not* a *transcendental* (“constitutive”) category of understanding <*Verstand*>.
2) What links Kant and Cantor is the deep comprehension that the Absolute can never be given as a whole. *The Whole* is always transcendent, and this is the main lesson of Kant’s antinomies: they arise, if knowledge wants to *transcend all possible experience*.
3) However, like Greek classics, Kant still holds that *completeness* belongs to the Whole, although it is slipping away from understanding into infinity: “Yet the idea of this completeness still lies in reason, irrespective of the possibility or impossibility of connecting empirical concepts to it adequately” (Kant, *Critique* B 444). – Not the world, the *reason* is whole.
4) In the modern cosmology, Kant’s critique (negative and positive) has to be applied to the concept of *multiverse* instead to the whole (or to “totality”) of *our* universe, which is yet *within* our possible experience, while the “whole *multi*verse” (whatever it means) is again the *Uni*verse which *transcends* all possible experience and therefore rises antinomies.