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INVERSE ESTIMATION OF EXPONENTIAL AND PIECE-WISE LINEAR APPROXIMATIONS OF THE HARDENING CURVE FROM THE TENSION TEST

I. Grešovnik
Centre for Computational Continuum Mechanics,
Ljubljana, Slovenia

T. Rodič
Faculty of Natural Sciences and Technology,
University of Ljubljana, Slovenia

D. R. J. Owen
Department of Civil Engineering,
University of Wales, Swansea, U.K.

Summary

An inverse numerical technique for estimation of exponential and piece-wise linear approximations of the hardening curve is considered. The technique is characterized by the finite element method which is employed in an iterative procedure to minimize the difference between measured and calculated values in a least-square sense. The technique is applied to the tension test.

1. INTRODUCTION

The tension test (Figure 1) is widely used for the mechanical testing of materials. However, accurate estimation of plastic material properties is difficult due to the non-uniform stress/strain distribution in the necking zone[1]. Bacause of this phenomenon, it is not possible to determine the hardening parameters directly by measuring elongations at different loads. In order to determine true stress the Bridgman correction[2] is often applied which requires additional measurements of contractions at the narrowest part of the deformed sample and curvature of the neck. The approach is based on the assumptions that the contour of the neck is the arc of a circle and that strains are constant over the cross section of the neck.

In this paper an inverse approach[3-8] for the estimation of hardening parameters is considered. This approach does not require additional measurements at the necking zone and does not incorporate Bridgman's idealisations.

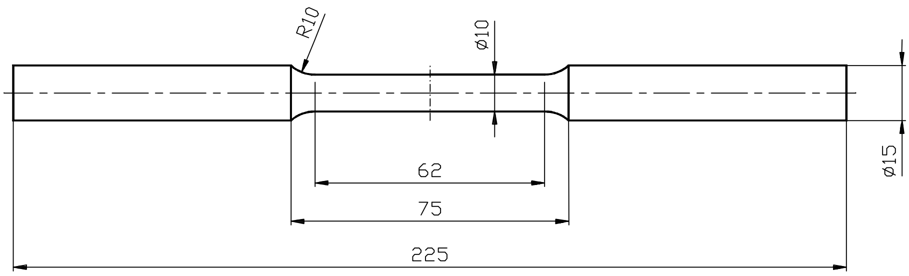

Figure 1. Sample geometry.

2. EXPONENTIAL APPROXIMATION

An exponential hardening law is assumed to approximate the relationship between the effective stress and effective strain:

$$\bar{\sigma} = C\bar{\varepsilon}^n \ . \tag{1}$$

The unknown parameters, C and n need to be derived from measured forces at certain elongations of the samples. Two series of measurements were performed for two different steel grades. The geometry of the samples is shown in Figure 1, while the experimental data are given in Tables 1 and 2 for each series. Graphic presentation of the same data for the first sample of each series is given in Figure 2.

Table 1. Experimental data for the first series.

Elongation [mm]	Force [N], sample 1	Force [N], sample 2	Force [N], sample 3
3	65900	68800	66800
4	67800	69900	67800
5	68650	70600	68700
6	68900	70600	68700
7	68850	69200	68400
8	68000	66600	68200
9	65800	61300	65100
10	61800	54100	59300

Table 2. Experimental data for the second series.

Elongation [mm]	Force [N], sample 1	Force [N], sample 2	Force [N], sample 3
3	86000	85800	84700
4	87500	86300	85600
5	87800	86500	86400
6	86500	85900	84500
7	81700	84600	80900
8	74800	78200	72600

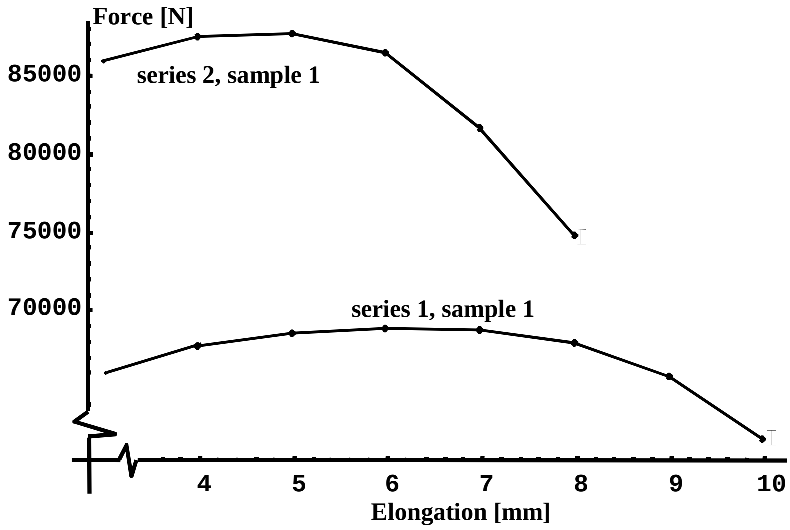

Figure 2. Masured data for the first sample of each series.

Solution of the problem was found by searching for the parameters which give the best agreement between measured and respective numerically calculated quantities. The agreement can be defined in different ways, but most commonly used is the least-square concept, mostly because of its statistical background[9,10]. The problem is solved by minimizing the function

$$\chi^2(C,n) = \sum_{i=1}^{N} \frac{\left(F_i^{(m)} - F_i(C,n)\right)^2}{\sigma_i^2} \ , \qquad (2)$$

where $F_i^{(m)}$ are measured forces at different elongations; $F_i(C,n)$ are the respective quantities calculated with the finite element model by assuming trial values of parameters C and n; σ_i are the expected errors of appropriate measurements and N is the number of measurements.

2.1 Results

The scatter of experimental data for the same series which is evident from Tables 1 and 2 is mainly due to differences in samples rather than experimental errors. This has an effect on the estimated parameters C and n. The results are summarized in Tables 3 and 4.

Table 3. Calculated parameters C and n for the first series.

	sample 1	sample 2	sample 3
$C[MPa]$	1271	1250	1258
n	0.1186	0.1010	0.1132

Table 4. Calculated parameters C and n for the second series.

	sample 1	sample 2	sample 3
$C[MPa]$	1492	1511	1462
n	0.08422	0.09269	0.08318

It seems that the applied numerical model simulates bahaviour of the investigated material adequately. This is indicated[9,10] by the fact that the obtained values of function $\chi^2(C,n)$ at its minimum were never much greater than one, assuming that the measurement errors σ_i in Equation 2 are one percent of the related measured values.

3. PIECE-WISE LINEAR APPROXIMATION

The flow stress of the material is a result of different hardening and softening phenomena which interact during plastic deformation. This interaction is often so complex that it is difficult to predict the form of the hardening curve $\overline{\sigma}(\overline{\varepsilon})$. In such cases it would be desirable to find an approximation of the hardeining curve without making any preassumptions about it. This can be done in several ways. In this paper, an approach where points of the hardening curve defining a piece-wise linear approximation are sought is considered.

The experimental measurements used for estimation of the piece-wise linear approximations are summarized in Table 5 and Figure 3. The data are for the first sample of the first series, but with 16 measurements instead of 8 used for evaluation of exponential approximation.

Table 5. Experimental data used to obtain a piece-wise linear approximation of the hardening curve.

Elongation [mm]	Force [N]
2	62200
2.5	64400
3	65900
3.5	67000
4	67800
4.5	68200
5	68650
5.5	68800
6	68900
6.5	69000
7	68850

7.5	68600
8	68000
8.5	67100
9	65800
9.5	64000

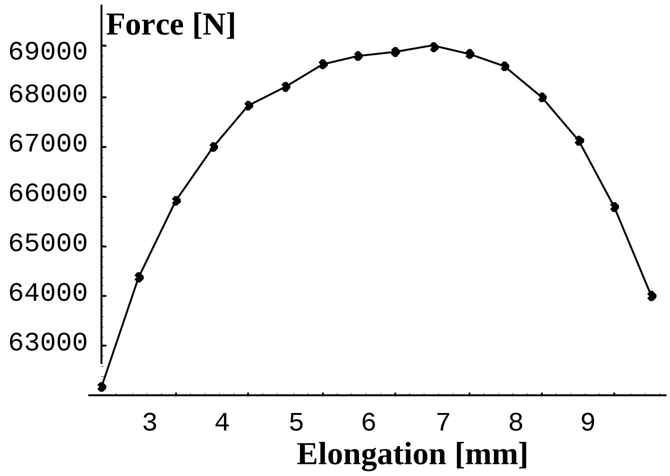

Figure 3. Measurements used for calculating a piecewise approximation of the hardening curve (measurements are for the first sample of the first series).

The points on the hardening curve were obtained by minimizing the function

$$\chi^2(\overline{\sigma}_1,\overline{\sigma}_2,...,\overline{\sigma}_M)=\sum_{i=1}^{N}\frac{\left(F_i^{(m)}-F_i(\overline{\sigma}_1,\overline{\sigma}_2,...,\overline{\sigma}_M)\right)^2}{\sigma_i^2}, \quad (3)$$

where parameters $\overline{\sigma}_i$ are values of the curve $\overline{\sigma}(\overline{\varepsilon})$ at arbitrary equivalent strains $\overline{\varepsilon}_i$. Yield stress was known from experiments.

3.1 Results

A piecewise-linear approximations of the hardening curve were calculated for the first sample of the first series, with 4, 6, 8 and 10 points. The results are shown in Figures 4 to 7. The exponential harening curve with parameters $C = 1271 M\ Pa$ and $n = 0.1186$ (as obtained by the inverse analysis assuming the exponential hardening law) is drawn in each figure for comparison. It is evident from these graphs that calculated piecewise linear approximations are in relatively good agreement with the calculated exponential approximation.

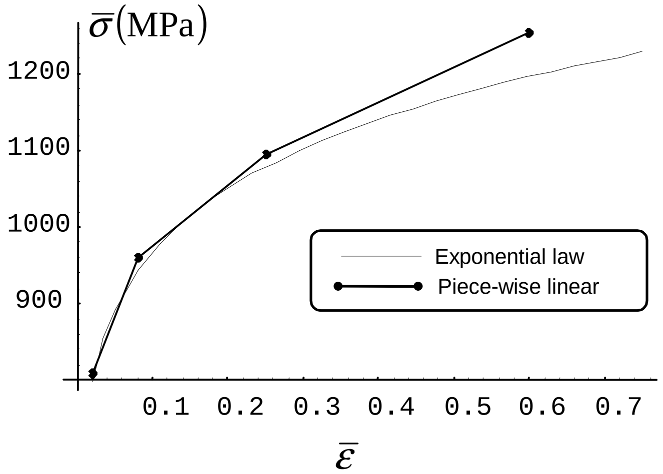

Figure 4. Comparison between exponential and piece-wise linear (4 points) approximations of the hardening curve.

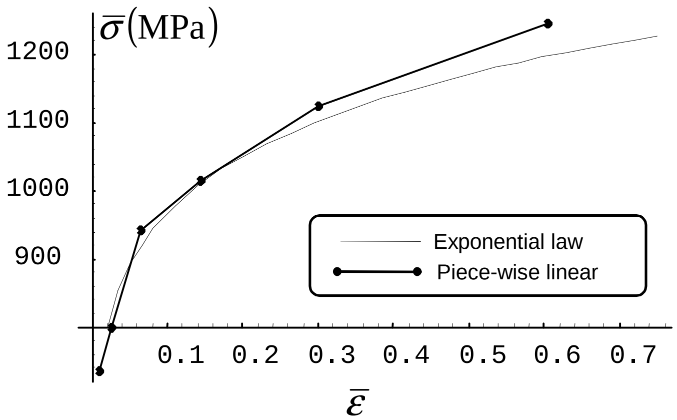

Figure 5. Comparison between exponential and piece-wise linear (6 points) approximations of the hardening curve.

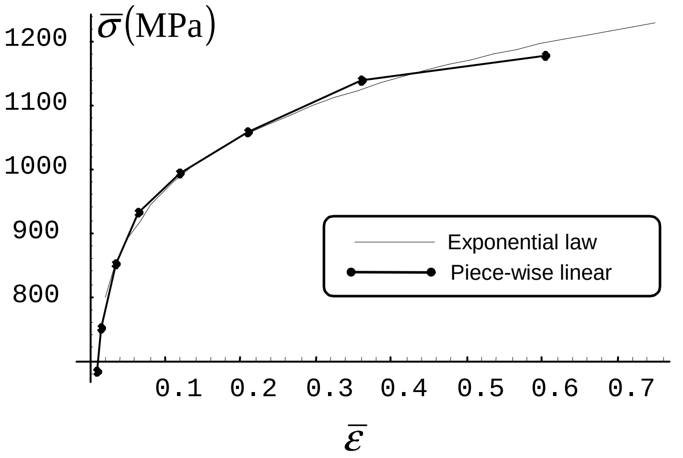

Figure 6. Comparison between exponential and piece-wise linear (8 points) approximations of the hardening curve.

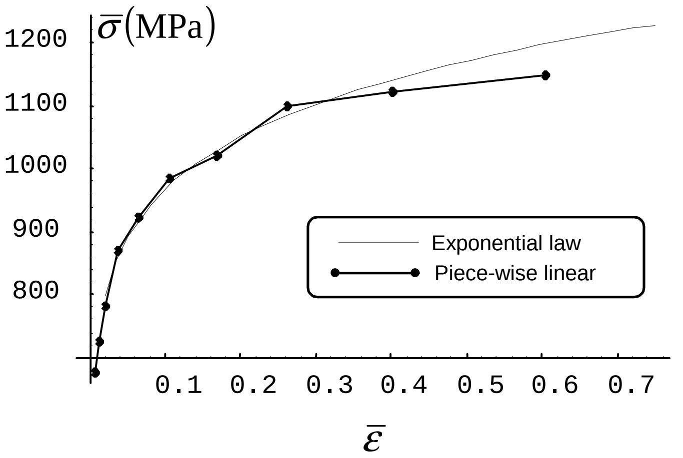

Figure 7. Comparison between exponential and piece-wise linear (10 points) approximations of the hardening curve.

4. NUMERICAL TESTS

A number of numerical tests were performed to investigate the stability and uniquenes of the inverse solutions for the exponential approximation of the hardening curve.

Several inverse analyses were performed with very different initial guesses and they always converged to the same results. This is the first indication that the problem is not ill-posed. Further examination was made by plotting the χ^2 function (Figures 8 and 9). It is evident from these figures that this function has a distinctive global minimum without local oscillations in its vicinity.

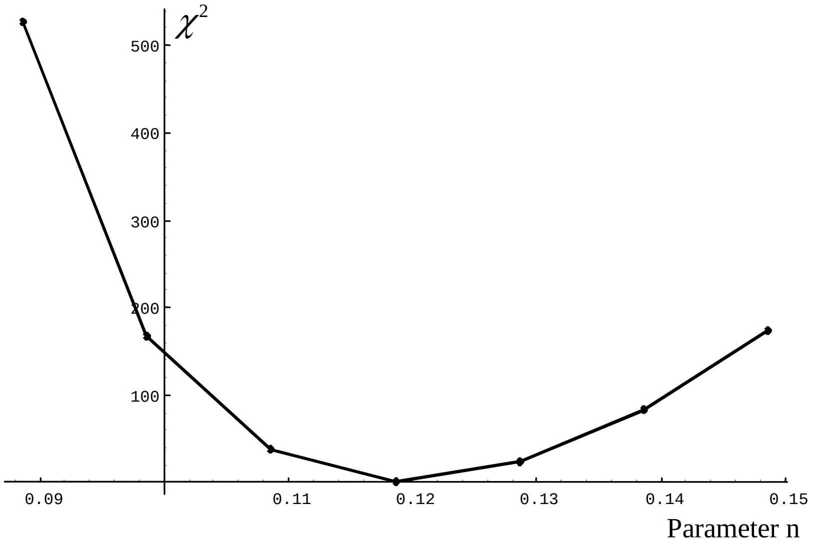

Figure 8. Dependence of function χ^2 *on parameter n at measured data for sample 1 of series 1. Parameter C is set to 1271 MPa.*

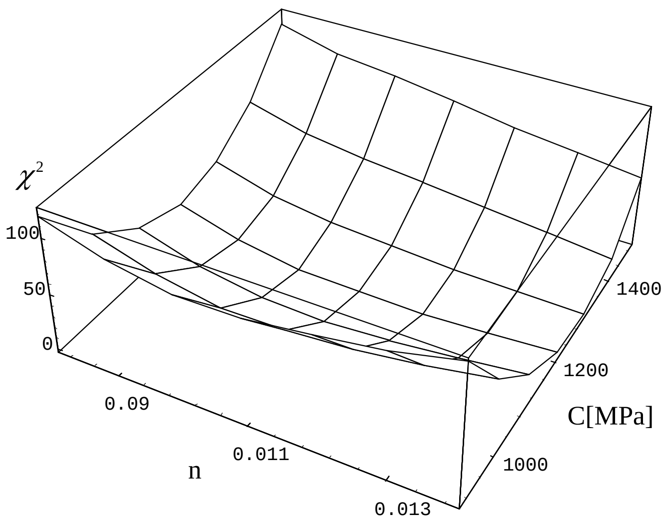

Figure 9. Dependence of function χ^2 *on both parameters for the same measured data as in Figure 2.*

To test the stability of the solutions, a Monte Carlo simulation[11] was performed. It was assumed that the correct values of both parameters were known. For this purpose, the previously calculated values for the first sample of the first series were taken, namely $C = 1271\ MPa$ and $n = 0{,}1186$ (see Table 1). With these values, so called "exact measurements" $F_i^{(0)}$ were obtained with the same finite element model used for the inverse analysis of the real measurements. The "simulated measurements" $F_i^{(m)}$ were successively obtained by adding random errors r_i to the "exact measurements". Errors were distributed normally as

$$\frac{d\,P}{d\,r_i} = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{r_i^2}{2\,s_i^2} \right) \ , \tag{4}$$

where s_i is the standard deviation of distribution. This distribution is often used to simulate measurement errors which do not have a clearly defined origin[10] .

For each set of "simulated measurements" parameters C and n were calculated. Three different sets of s_i were chosen so that ratios

$$R_i = \frac{s_i}{\left| F_i^0 \right|} \tag{5}$$

were uniform within each set. Fifty numerical experiments were performed for $R = 0.01$, twenty for $R = 0.1$ and twenty for $R = 0.001$. Then average values $\bar{z}$ and dispersions S_z of the searched parameters were calculated for each set, according to

$$\bar{z} = \frac{1}{k} \sum_{i=1}^{k} z_i \tag{6}$$

and

$$S_z^2 = \frac{1}{k-1} \sum_{i=1}^{k} \left(z_i - \bar{z} \right)^2 \ . \tag{7}$$

The results are summarized in Table 10. Figure 10 shows the whole distribution of calculated parameters at $R = 0.01$.

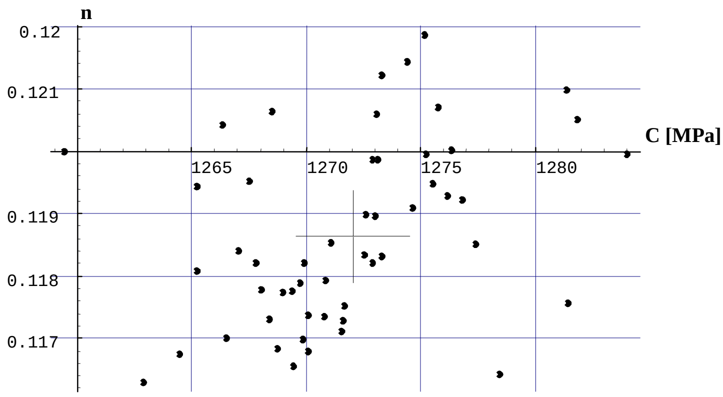

Figure 10. Results of the Monte carlo simulation for $R = 0.01$.

Table 6. Results of Monte Carlo simulations: Average values and dispersions of calculated parameters at different R_i.

	$R = 0{,}001$	$R = 0{,}01$	$R = 0{,}1$
$\overline{C}$	1271.4	1271.8	1287
S_C	0.58	4.9	69
$\overline{n}$	0.118628	0.11867	0.1163
S_n	0.00016	0.0015	0.014

5. CONCLUSION

An inverse estimation of the exponential and piece-wise linear approximations of the hardening curve from the tension test is presented. Force measurements at different elongations were used as input to the inverse analysis. The solution system is implemented as a computational shell arround the Elfen finite element[12] code. An important advantage of this approach is that parameters are derived by using the same numerical model which is then applied in direct simulations.

Numerical tests show that the evaluation of the exponential approximation is well conditioned, that no problems with uniquenes of the solutions were present and that it provides better fit then all piece-wise linear approximations considered in this paper. This suggests that exponential law describes characteristics of the investigated material well.

Acknowledgment

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