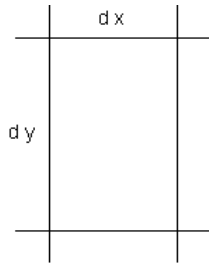
Chapter 13

Mercator Charts and Plotting Sheets

Sophisticated navigation is almost impossible without the use of a map (chart), a projection of a certain area of the earth's surface on a plane sheet of paper. There are several types of map projection, but the **Mercator projection,** named after the German cartographer Gerhard Kramer (Latin: Gerardus Mercator), is mostly used in navigation because it produces charts with an orthogonal grid which is most convenient for measuring directions and plotting lines of position. Further, rhumb lines appear as straight lines on a Mercator chart. Great circles do not, apart from meridians and the equator which are also rhumb lines.

In order to construct a Mercator chart, we have to remember how the grid printed on a globe looks. At the equator, an area of, e. g., 5 by 5 degrees looks almost like a square, but it becomes an increasingly narrow trapezoid as we move toward one of the poles. While the distance between two adjacent parallels of latitude remains constant, the distance between two meridians becomes progressively smaller as the latitude increases. An area with the infinitesimally small dimensions dLat and dLon would appear as an oblong with the dimensions dx and dy on our globe (*Fig. 13-1*):

Fig. 13-1



$$dx = c' \cdot d\,Lon \cdot \cos Lat$$

$$dy = c' \cdot d\,Lat$$

dx contains the factor cos Lat since the circumference of a parallel of latitude is in direct proportion to cos Lat. The constant c' is the scale of the globe (measured in, e. g., mm/°).

Since we require **any** rhumb line to appear as a **straight** line intersecting all meridians at a constant angle, meridians have to be equally spaced vertical lines on our chart, and an infinitesimally small oblong defined by dLat and dLon must have a constant aspect ratio, regardless of its position on the chart (dy/dx = const.).

Therefore, if we transfer the oblong defined by dLat and dLon from the globe to our chart, we get the dimensions:

$$dx = c \cdot d\,Lon$$

$$dy = c \cdot \frac{d\,Lat}{\cos Lat}$$

The new constant c is the scale of the chart. Now, dx remains constant (parallel meridians) but dy is a function of the latitude at which our small oblong is located. To obtain the smallest distance between any point at the latitude Lat_P and the equator, we integrate:

$$Y = \int_0^Y dy = c \cdot \int_0^{Lat_P} \frac{d\,Lat}{\cos\,Lat} = c \cdot \ln \tan\left(\frac{Lat_P}{2} + \frac{\pi}{4}\right)$$

Y is the distance of the respective parallel of latitude from the equator. In the above equation, angles are given in circular measure (radians). If we measure angles in degrees, the equation is stated as:

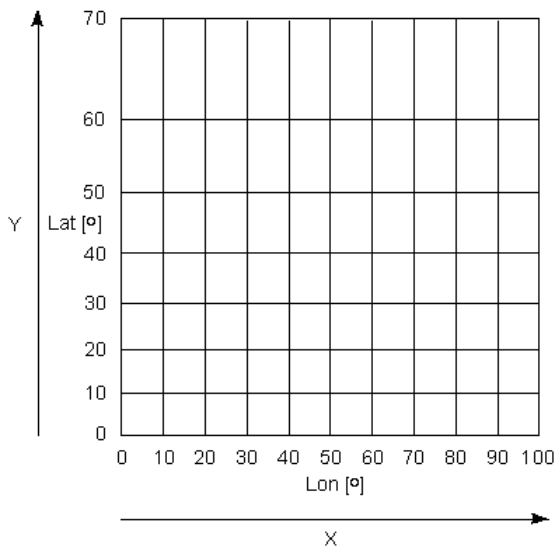
$$Y = c \cdot \ln \tan\left(\frac{Lat_P[°]}{2} + 45° \right)$$

The distance of any point from the Greenwich meridian (Lon = 0°) varies proportionally with the longitude of the point, Lon_P. X is the distance of the respective meridian from the Greenwich meridian:

$$X = \int_0^{Lon_P} dx = c \cdot Lon_P$$

Fig. 13-2 shows an example of the resulting grid. While meridians of longitude appear as equally spaced vertical lines, parallels of latitude are horizontal lines drawn farther apart as the latitude increases. Y would be infinite at 90° latitude.

Fig. 13-2



Mercator charts have the disadvantage that geometric distortions increase as the distance from the equator increases. The Mercator projection is therefore not suitable for polar regions. A circle of equal altitude, for example, would appear as a distorted ellipse at higher latitudes. Areas near the poles, e. g., Greenland, appear much greater on a Mercator map than on a globe.

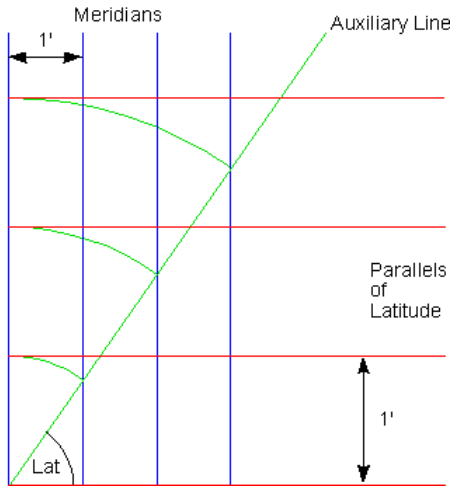
It is often said that a Mercator chart is obtained by projecting each point of the surface of a globe along lines radiating from the center of the globe to the inner surface of a hollow cylinder tangent to the globe at the equator. This is only a rough approximation. As a result of such a projection, Y would be proportional to tan Lat, and the aspect ratio of a small oblong defined by dLat and dLon would vary, depending on its position on the chart.

If we magnify a small part of a Mercator chart, e. g., an area of 30' latitude by 40' longitude, we will notice that the spacing between the parallels of latitude now seems to be almost constant. An approximated Mercator grid of such a small area can be constructed by drawing equally spaced horizontal lines, representing the parallels of latitude, and equally spaced vertical lines, representing the meridians. The spacing of the parallels of latitude, Δy, defines the scale of our chart, e. g., 5mm/nm. The spacing of the meridians, Δx, is a function of the middle latitude, Lat_M, the latitude represented by the horizontal line going through the center of our sheet of paper:

$$\Delta x = \Delta y \cdot \cos Lat_M$$

A sheet of paper with such a simplified Mercator grid is called a **small area plotting sheet** and is a very useful tool for plotting lines of position (*Fig. 13-3*).

Fig. 13-3



If a calculator or trigonometric table is not available, the meridian lines can be constructed with the following graphic method:

We take a sheet of blank paper and draw the required number of equally spaced horizontal lines (parallels). A spacing of 3 - 10 mm per nautical mile is recommended for most applications.

We draw an auxiliary line intersecting the parallels of latitude at an angle equal to the mid latitude. Then we mark the map scale, e.g., 5 mm/nm, periodically on this line, and draw the meridian lines through the points thus located (*Fig. 13-3*). Compasses can be used to transfer the map scale to the auxiliary line.

Small area plotting sheets are available at nautical book stores.

A useful program (shareware) for printing small area plotting sheets for any given latitude between 0° and 80° can be downloaded from this web site:

http://perso.easynet.fr/~philimar/graphpapeng.htm (present URL)