Chapter 4

Finding One's Position (Sight Reduction)

Lines of Position

Any geometrical or physical line passing through the observer's (still unknown) position and accessible through measurement or observation is called a **line of position**, **LoP**. Examples are circles of equal altitude, meridians of longitude, parallels of latitude, bearing lines (compass bearings) of terrestrial objects, coastlines, rivers, roads, or railroad tracks. **A single LoP indicates an infinite series of possible positions. The observer's actual position is marked by the point of intersection of at least two LoP's, regardless of their nature.** The concept of the position line is essential to modern navigation.

Sight Reduction

Deriving a line of position from altitude and GP of a celestial object is called **sight reduction** in navigator´s language. Understanding the process completely requires some background in spherical trigonometry, but knowing the basic concepts and a few equations is sufficient for most applications of celestial navigation. The theoretical explanation, using the **law of cosines** and the **navigational triangle**, is given in chapter 10 and 11. In the following, we will discuss the semi-graphic methods developed by *Sumner* and *St. Hilaire*. Both methods require relatively simple calculations only and enable the navigator to plot lines of position on a navigation chart or **plotting sheet** (see chapter 13).

Knowing altitude and GP of a body, we also know the radius of the corresponding circle of equal altitude (our line of position) and the location of its center. As mentioned in chapter 1 already, plotting circles of equal altitude directly on a chart is usually impossible due to their large dimensions and the distortions caused by map projection. However, *Sumner* and *St. Hilaire* showed that only a small arc of each circle of equal altitude is needed to find one's position. Since this arc is comparatively short, it can be replaced with a secant or tangent a of the circle.

The Intercept Method

This is the most versatile and most popular sight reduction procedure. In the second half of the 19th century, the French navy officer and later admiral *St. Hilaire* found that a straight line **tangent** to the circle of equal altitude in the vicinity of the observer's position can be utilized as a line of position. The procedure comprises the following steps:

1.

First, we need an **initial position** which should be less than ca. 100 nm away from our actual (unknown) position. This may be our **estimated position**, our **dead reckoning position**, **DRP** (chapter 11), or an **assumed position**, **AP**. We mark this position on our navigation chart or **plotting sheet** (chapter 13) and note the corresponding latitude and longitude. An assumed position is a chosen point in the vicinity of our estimated position or DRP, preferably the nearest point on the chart where two grid lines intersect. An assumed position is sometimes preferred since it may be more convenient for plotting lines and measuring angles on the plotting sheet. Some sight reduction tables are based upon AP's because they require integer values for coordinates. **The following procedures and formulas refer to an AP. They would be exactly the same, however, when using a DRP or an estimated position.**

2.

Using the laws of spherical trigonometry (chapter 10 and 11), we calculate the altitude of the observed body as it would appear at our AP (reduced to the celestial horizon). This altitude is called **calculated** or **computed altitude**, **Hc**:

$$Hc = \arcsin\left(\sin Lat_{AP} \cdot \sin Dec + \cos Lat_{AP} \cdot \cos Dec \cdot \cos LHA\right)$$

or

$$Hc = \arcsin\left(\sin Lat_{AP} \cdot \sin Dec + \cos Lat_{AP} \cdot \cos Dec \cdot \cos t\right)$$

Lat_{AP} is the **geographic latitude** of AP. **Dec** is the declination of the observed body. **LHA** is the **local hour angle** of the body, the angular distance of GP westward from the local meridian going through AP, measured from 0° through 360°.

$$LHA = \begin{cases} GHA + Lon_{AP} & \text{if} \quad 0° \le GHA + Lon_{AP} \le 360° \\ GHA + Lon_{AP} + 360° & \text{if} \quad GHA + Lon_{AP} < 0° \\ GHA + Lon_{AP} - 360° & \text{if} \quad GHA + Lon_{AP} > 360° \end{cases}$$

Instead of the local hour angle, we can use the **meridian angle**, **t**, to calculate Hc. Like LHA, t is the algebraic sum of GHA and Lon_{AP}. In contrast to LHA, however, t is measured westward (0°...+180°) or eastward (0°...–180°) from the local meridian:

$$t = \begin{cases} GHA + Lon_{AP} & \text{if} \quad GHA + Lon_{AP} \le 180° \\ GHA + Lon_{AP} - 360° & \text{if} \quad GHA + Lon_{AP} > 180° \end{cases}$$

Lon_{AP} is the **geographic longitude** of AP. **The sign of Lon_{AP} has to be observed carefully (E:+, W:–).**

3.

We calculate the **azimuth** of the body, **Az_N**, the direction of GP with reference to the geographic north point on the horizon, measured clockwise from 0° through 360° at AP. We can calculate the azimuth either from Hc (altitude azimuth) or from LHA or t (time azimuth). Both methods give identical results.

The formula for the altitude azimuth is stated as:

$$Az = \arccos \frac{\sin Dec - \sin Hc \cdot \sin Lat_{AP}}{\cos Hc \cdot \cos Lat_{AP}}$$

The **azimuth angle**, **Az**, the angle formed by the meridian going through AP and the great circle going through AP and GP, is not necessarily identical with **Az_N** since the arccos function yields results between 0° and +180°. To obtain Az_N, we apply the following rules:

$$Az_N = \begin{cases} Az & \text{if} \quad \sin LHA \le 0 \quad (\text{or } \ t \le 0) \\ 360° - Az & \text{if} \quad \sin LHA > 0 \quad (\text{or } \ t > 0) \end{cases}$$

The formula for the time azimuth is stated as:

$$Az = \arctan \frac{-\sin LHA}{\cos Lat_{AP} \cdot \tan Dec - \sin Lat_{AP} \cdot \cos LHA}$$

Again, the meridian angle, t, may be substituted for LHA. Since the arctan function returns results between -90° and +90°, the time azimuth formula requires a different set of rules to obtain Az_N:

$$Az_N = \begin{cases} Az & \text{if} \quad \text{numerator} > 0 \quad \text{AND} \quad \text{denominator} > 0 \\ Az + 360° & \text{if} \quad \text{numerator} < 0 \quad \text{AND} \quad \text{denominator} > 0 \\ Az + 180° & \text{if} \quad \text{denominator} < 0 \end{cases}$$

Fig. 4-1 illustrates the angles involved in the calculation of Hc (= 90°-z) and Az:

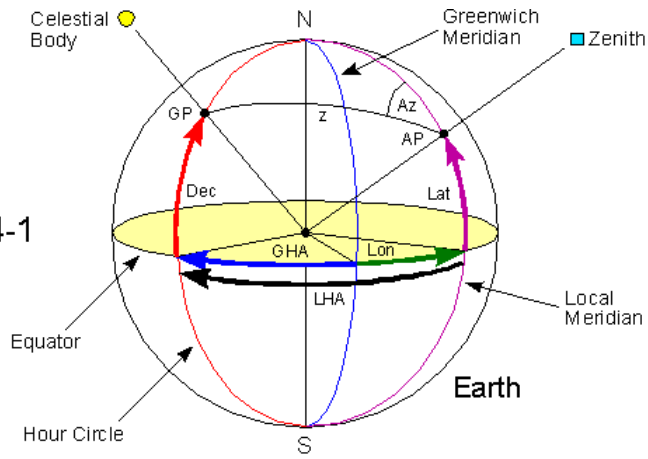


Fig. 4-1

The above formulas are derived from the **navigational triangle** formed by N, AP, and GP. A detailed explanation is given in chapter 11. Mathematically, the calculation of Hc and Az_N is a transformation of equatorial coordinates to horizontal coordinates.

4.

We calculate the **intercept**, **Ic**, the difference between observed altitude, Ho, (chapter 2) and computed altitude, Hc. For the following procedures, the intercept, which is directly proportional to the difference between the radii of the corresponding circles of equal altitude, is expressed in distance units:

$$Ic[nm] = 60 \cdot (Ho[°] - Hc[°]) \qquad \text{or} \qquad Ic[km] = \frac{40031.6}{360} \cdot (Ho[°] - Hc[°])$$

(The mean perimeter of the earth is 40031.6 km.)

When going the distance Ic along the azimuth line from AP toward GP (Ic > 0) or away from GP (Ic < 0), we reach the circle of equal altitude for our actual position (LoP). As shown in *Fig. 4-2*, a straight line perpendicular to the azimuth line and tangential to the circle of equal altitude for the actual position is a fair approximation of our circular LoP as long as we stay in the vicinity of our position.

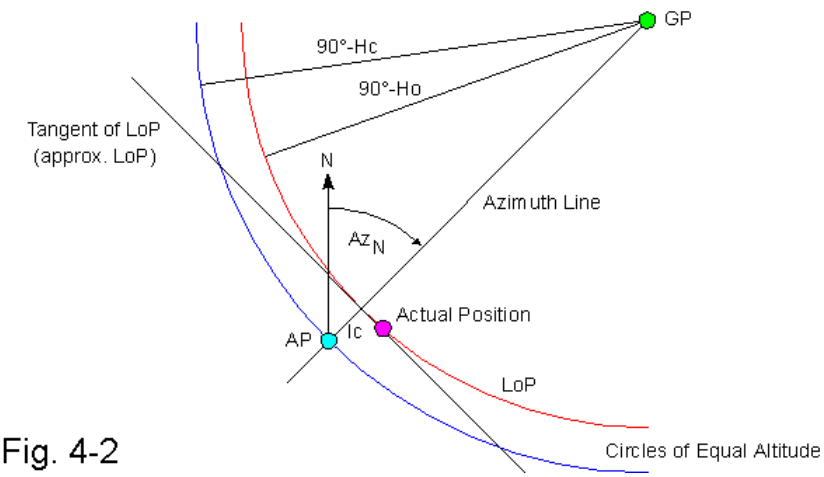
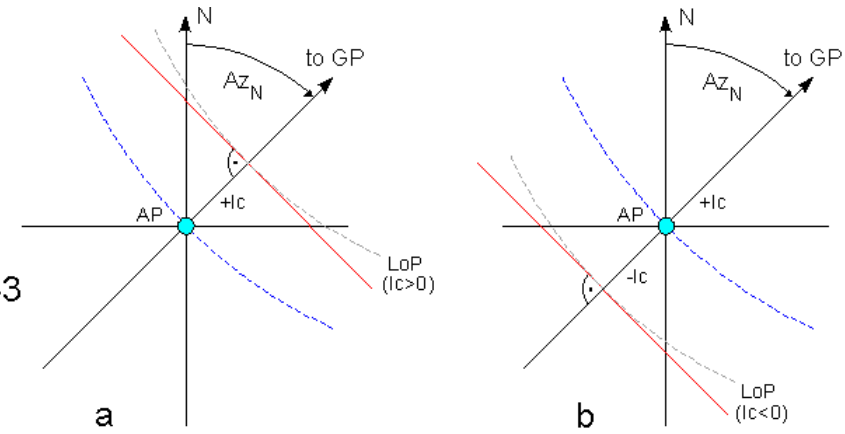


Fig. 4-2

5.

We take the chart and draw a suitable part of the azimuth line through AP. We measure the intercept, Ic, along the azimuth line (towards GP if Ic>0, away from GP if Ic<0) and draw a perpendicular through the point thus located. This perpendicular is our approximate line of position (*Fig. 4-3*).

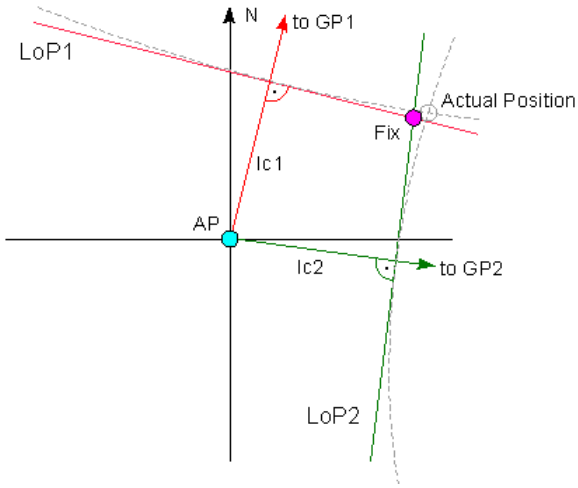
Fig. 4-3



6.

To obtain the second LoP needed to find our position, we repeat the procedure (same AP) with altitude and GP of a second celestial body or the same body at a different time of observation (*Fig. 4-4*). The point where both LoP's (tangents) intersect is our **improved position**. In navigator's language, the position thus located is called **fix**.

Fig. 4-4

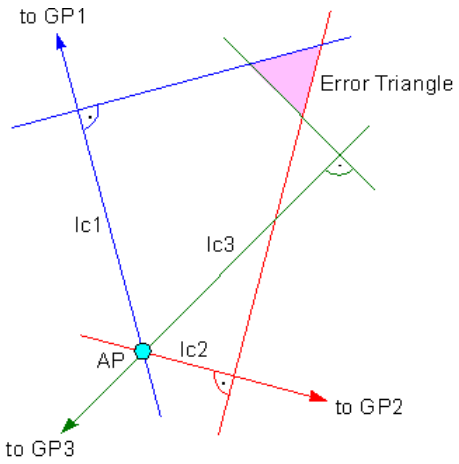


The intercept method ignores the curvature of the actual LoP's. The resulting error remains tolerable as long as the radii of the circles of equal altitude are great enough and AP is not too far from the actual position (see chapter 16). The geometric error inherent to the intercept method can be decreased by **iteration**, i.e., substituting the position thus obtained for AP and repeating the calculations (same altitudes and GP's). This will result in a more accurate position. If necessary, we can reiterate the procedure until the obtained position remains virtually constant.

Since a dead reckoning position is usually nearer to our true position than an assumed position, the latter may require a greater number of iterations.

Accuracy is also improved by observing three bodies instead of two. Theoretically, the LoP’s should intersect each other at a single point. Since no observation is entirely free of errors, we will usually obtain three points of intersection forming an **error triangle** (*Fig. 4-5*).

Fig. 4-5



Area and shape of the triangle give us a rough estimate of the quality of our observations (see chapter 16). Our **most probable position**, **MPP**, is usually in the vicinity of the center of the inscribed circle of the triangle (the point where the bisectors of the three angles meet).

When observing more than three bodies, the resulting LoP's will form the corresponding polygons.

Direct Computation

If we do not want to plot our lines of position to determine our fix, we can find the latter by computation. Using the **method of least squares**, it is possible to calculate the most probable position directly from an unlimited number, n, of observations (n > 1) without the necessity of a graphic plot. The Nautical Almanac provides the following procedure. First, the auxiliary quantities A, B, C, D, E, and G have to be calculated:

$$A = \sum_{i=1}^{n} \cos^2 Az_i \qquad B = \sum_{i=1}^{n} \sin Az_i \cdot \cos Az_i \qquad C = \sum_{i=1}^{n} \sin^2 Az_i$$

$$D = \sum_{i=1}^{n} Ic_i \cdot \cos Az_i \qquad E = \sum_{i=1}^{n} Ic_i \cdot \sin Az_i \qquad G = A \cdot C - B^2$$

The geographic coordinates of the observer's MPP are then obtained as follows:

$$Lon = Lon_{AP} + \frac{A \cdot E - B \cdot D}{G \cdot \cos Lat_{AP}} \qquad Lat = Lat_{AP} + \frac{C \cdot D - B \cdot E}{G}$$

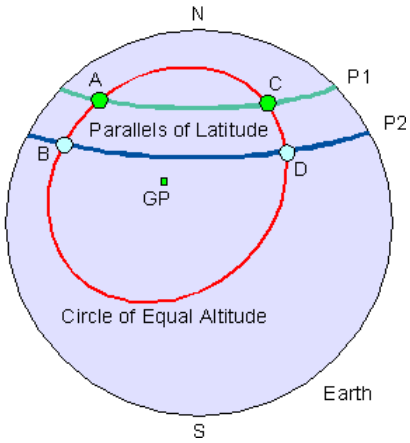
The method does not correct the geometric errors inherent to the intercept method. These are eliminated, if necessary, by applying the method iteratively until the MPP remains virtually constant. The N.A. suggests repeating the calculations if the obtained MPP is more than 20 nautical miles from AP or the initial estimated position.

Sumner's Method

This sight reduction procedure was discovered by *T. H. Sumner*, an American sea captain, in the first half of the 19th century. Although it is rarely used today, it is still an interesting alternative to *St. Hilaire's* intercept method. The theoretical explanation is given in chapter 11 (navigational triangle).

Sumner had the brilliant idea to derive a line of position from the points where a circle of equal altitude intersects two chosen parallels of latitude, P1 and P2 (*Fig. 4-6*).

Fig. 4-6



An observer being between the parallels P1 and P2 is either on the arc A-B or on the arc C-D. With an estimate of his longitude, the observer can easily find on which of both arcs he is, for example, A-B. The arc thus found is the relevant part of his line of position, the other arc is discarded. We can approximate the LoP by drawing a straight line through A and B which is a secant of the circle of equal altitude. This secant is called **Sumner line**. Before plotting the Sumner line on our chart, we have to find the longitudes of the points of intersection, A, B, C, and D. This is the procedure:

1.

We choose a parallel of latitude (P1) north of our estimated latitude. Preferably, the assumed latitude, Lat, should refer to the nearest grid line on our chart or plotting sheet.

2.

Solving the altitude formula (see above) for t and substituting Ho for Hc, we get:

$$t = \pm \arccos \frac{\sin Ho - \sin Lat \cdot \sin Dec}{\cos Lat \cdot \cos Dec}$$

Now, t is a function of latitude, declination, and the observed altitude of the body. Lat is the assumed latitude. In other words, the meridian angle of a body is either +t or –t when an observer being at the latitude Lat measures the altitude Ho. Using the following formulas, we obtain the longitudes which mark the points where the circle of equal altitude intersects the assumed parallel of latitude, for example, the points A and C if we choose P1:

$$Lon_1 = t - GHA$$

$$\text{If } Lon_1 < -180° \quad \rightarrow \quad Lon_1 + 360°$$

$$Lon_2 = 360° - t - GHA$$

$$\text{If } Lon_2 < -180° \quad \rightarrow \quad Lon_2 + 360°$$

$$\text{If } Lon_2 > +180° \quad \rightarrow \quad Lon_2 - 360°$$

Comparing the longitudes thus obtained with our estimated longitude, we select the relevant longitude and discard the other. This method of finding longitude is called **time sight** (see chapter 6).

3.

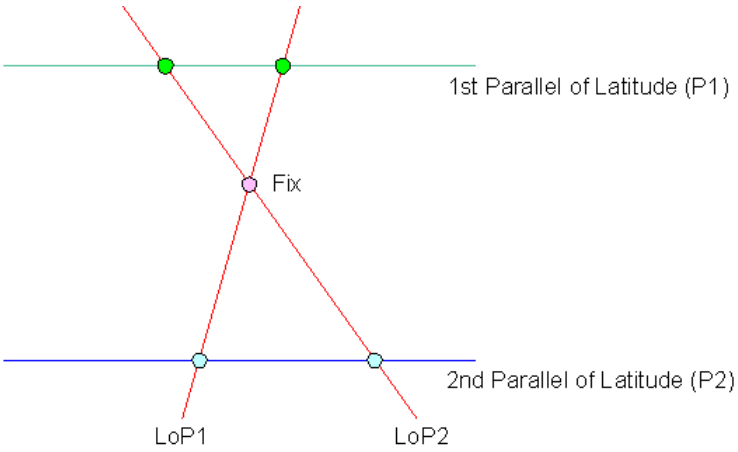
We chose a parallel of latitude (P2) south of our estimated latitude. The distance between P1 and P2 should not exceed 1 or 2 degrees. We repeat steps 1 and 2 with the second parallel of latitude, P2.

4.

On our plotting sheet, we mark each remaining longitude on the corresponding parallel and plot the Sumner line through the points thus located.

To obtain a fix, we repeat the above procedure with the same parallels and a second body. The point where both Sumner lines, LoP1 and LoP2, intersect is our fix (*Fig. 4-7*).

Fig. 4-7



If both assumed parallels of latitude are either north or south of our actual position, we will of course find the point of intersection outside the interval defined by both parallels. Nevertheless, a fix thus obtained is correct.

A fix obtained with *Sumner's* method, too, has a small error caused by neglecting the curvature of the circles of equal altitude. Similar to the intercept method, we can improve the fix by iteration. In this case, we choose a new pair of assumed latitudes, nearer to the fix, and repeat the whole procedure.

A Sumner line may be inaccurate under certain conditions (see time sight, chapter 6). Apart from these restrictions, *Sumner's* method is fully adequate. It has even the advantage that lines of position are plotted without a protractor.

As with the intercept method, we can plot Sumner lines resulting from three (or more) observations to obtain an error triangle (polygon).

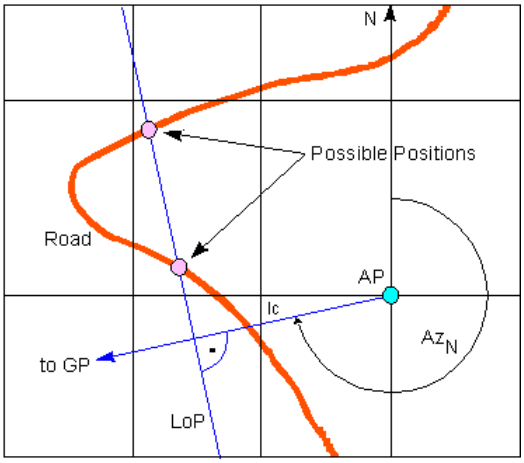
Sumner's method revolutionized celestial navigation and can be considered as the beginning of modern position line navigation which was later perfected by *St. Hilaire's* intercept method.

Combining Different Lines of Position

Since the point of intersection of any two LoP's, regardless of their nature, marks the observer's geographic position, one celestial LoP may suffice to find one's position if another LoP of a different kind is available.

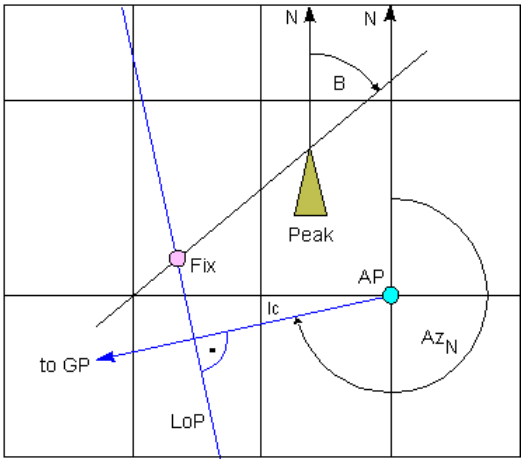
In the desert, for instance, we can determine our current position by finding the point on the map where a LoP obtained by observation of a celestial object intersects the dirt road we are traveling on (*Fig. 4-8*).

Fig. 4-8



We could as well find our position by combining our celestial LoP with the bearing line of a distant mountain peak or any other prominent landmark (*Fig. 4-9*). B is the compass bearing of the terrestrial object (corrected for magnetic declination).

Fig. 4-9



Both examples clearly demonstrate the versatility of position line navigation.