Chapter 1

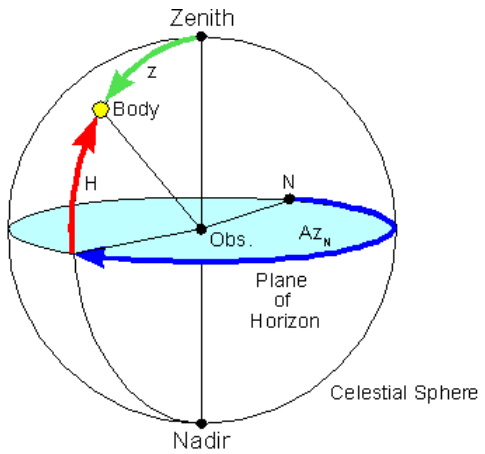
The Elements of Celestial Navigation

Celestial navigation, a branch of applied astronomy, is the art and science of finding one's geographic position through astronomical observations, particularly by measuring **altitudes** of celestial bodies – sun, moon, planets, or stars.

An observer watching the night sky without knowing anything about geography and astronomy might spontaneously get the impression of being on a plane located at the center of a huge, hollow sphere with the celestial bodies attached to its inner surface. Indeed, this naive model of the universe was in use for millennia and developed to a high degree of perfection by ancient astronomers. Still today, it is a useful tool for celestial navigation since the navigator, like the astronomers of old, measures **apparent positions** of bodies in the sky but not their absolute positions in space.

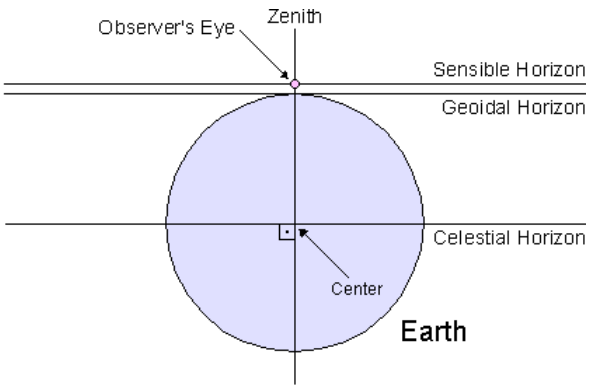
Following the above scenario, the apparent position of a body in the sky is defined by the **horizon system of coordinates**. In this system, the observer is located at the center of a fictitious hollow sphere of infinite diameter, the **celestial sphere**, which is divided into two hemispheres by the plane of the **celestial horizon** (*Fig. 1-1*). The **altitude**, **H**, is the vertical angle between the line of sight to the respective body and the celestial horizon, measured from 0° through +90° when the body is above the horizon (visible) and from 0° through -90° when the body is below the horizon (invisible). The **zenith distance**, **z**, is the corresponding angular distance between the body and the **zenith**, an imaginary point vertically overhead. The zenith distance is measured from 0° through 180°. The point opposite to the zenith is called **nadir** (z = 180°). H and z are **complementary angles** (H + z = 90°). The **azimuth**, $\mathbf{Az_N}$, is the horizontal direction of the body with respect to the geographic (true) north point on the horizon, measured clockwise from 0° through 360°.

Fig. 1-1



In reality, the observer is not located at the celestial horizon but at the the **sensible horizon.** *Fig. 1-2* shows the three horizontal planes relevant to celestial navigation:

Fig. 1-2



The **sensible horizon** is the horizontal plane passing through the observer's eye. The **celestial horizon** is the horizontal plane passing through the center of the earth which coincides with the center of the celestial sphere. Moreover, there is the **geoidal horizon,** the horizontal plane tangent to the earth at the observer's position. These three planes are parallel to each other.

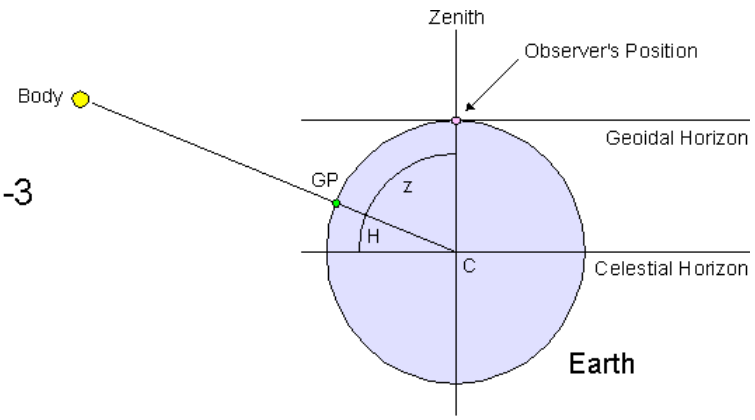
The sensible horizon merges into the geoidal horizon when the observer's eye is at sea or ground level. Since both horizons are usually very close to each other, they can be considered as identical under practical conditions. None of the above horizontal planes coincides with the **visible horizon**, the line where the earth's surface and the sky appear to meet.

Calculations of celestial navigation always refer to the geocentric altitude of a body, the altitude with respect to a fictitious observer being at the celestial horizon and at the center of the earth which coincides the center of the celestial sphere. Since there is no way to measure this altitude directly, it has to be derived from the altitude with respect to the visible or sensible horizon (altitude corrections, chapter 2).

A **marine sextant** is an instrument designed to measure the altitude of a body with reference to the visible sea horizon. Instruments with any kind of an **artificial horizon** measure the altitude referring to the sensible horizon (chapter 2).

Altitude and zenith distance of a celestial body depend on the distance between a terrestrial observer and the **geographic position of the body**, **GP**. GP is the point where a straight line from the body to the center of the earth, C, intersects the earth's surface (*Fig. 1-3*).

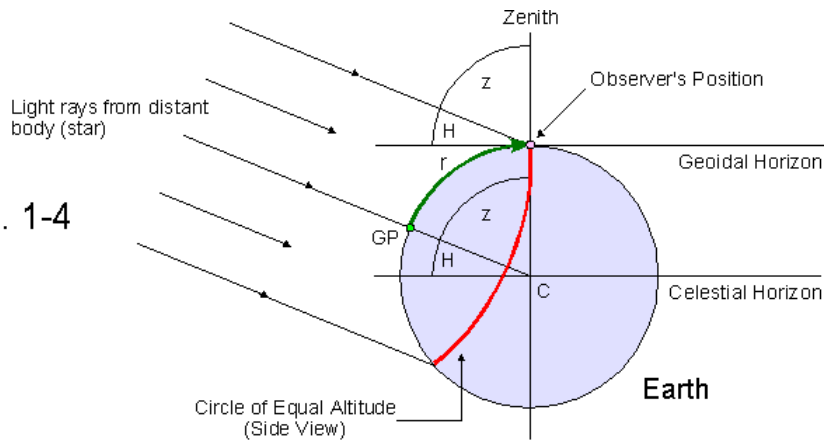
Fig. 1-3



A body appears in the zenith (z = 0°, H = 90°) when GP is identical with the observer's position. A terrestrial observer moving away from GP will observe that the altitude of the body decreases as his distance from GP increases. The body is on the celestial horizon (H = 0°, z = 90°) when the observer is one quarter of the circumference of the earth away from GP.

For a given altitude of a body, there is an infinite number of positions having the same distance from GP and forming a circle on the earth's surface whose center is on the line C–GP, below the earth's surface. Such a circle is called a **circle of equal altitude**. An observer traveling along a circle of equal altitude will measure a constant altitude and zenith distance of the respective body, no matter where on the circle he is. The radius of the circle, r, measured along the surface of the earth, is directly proportional to the observed zenith distance, z (*Fig 1-4*).

Fig. 1-4



$$r\,[nm] = 60 \cdot z\,[°] \qquad or \qquad r\,[km] = \frac{Perimeter\,of\ Earth\,[km]}{360°} \cdot z\,[°]$$

One nautical mile (1 nm = 1.852 km) is the **great circle distance** of one minute of arc (the definition of a great circle is given in chapter 3). The mean perimeter of the earth is 40031.6 km.

Light rays coming from distant objects (stars) are virtually parallel to each other when reaching the earth. Therefore, the altitude with respect to the geoidal (sensible) horizon equals the altitude with respect to the celestial horizon. In contrast, light rays coming from the relatively close bodies of the solar system are diverging. This results in a measurable difference between both altitudes (parallax). The effect is greatest when observing the moon, the body closest to the earth (see chapter 2, *Fig. 2-4*).

The azimuth of a body depends on the observer's position on the circle of equal altitude and can assume any value between 0° and 360°.

Whenever we measure the altitude or zenith distance of a celestial body, we have already gained partial information about our own geographic position because we know we are somewhere on a circle of equal altitude with the radius r and the center GP, the geographic position of the body. Of course, the information available so far is still incomplete because we could be anywhere on the circle of equal altitude which comprises an infinite number of possible positions and is therefore also called a **circle of position** (see chapter 4).

We continue our mental experiment and observe a second body in addition to the first one. Logically, we are on two circles of equal altitude now. Both circles overlap, intersecting each other at two points on the earth's surface, and one of those two points of intersection is our own position (*Fig. 1-5a*). Theoretically, both circles could be tangent to each other, but this case is highly improbable (see chapter 16).

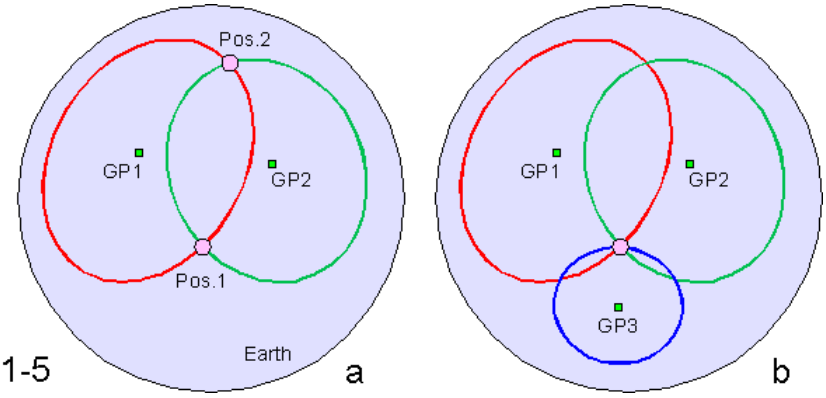


Fig. 1-5

In principle, it is not possible to know which point of intersection – Pos.1 or Pos.2 – is identical with our actual position unless we have additional information, e.g., a fair estimate of where we are, or the **compass bearing** of at least one of the bodies. Solving the problem of **ambiguity** can also be achieved by observation of a third body because there is only one point where all three circles of equal altitude intersect (*Fig. 1-5b*).

Theoretically, we could find our position by plotting the circles of equal altitude on a globe. Indeed, this method has been used in the past but turned out to be impractical because precise measurements require a very big globe. Plotting circles of equal altitude on a map is possible if their radii are small enough. This usually requires observed altitudes of almost 90°. The method is rarely used since such altitudes are not easy to measure. In most cases, circles of equal altitude have diameters of several thousand nautical miles and can **not** be plotted on usual maps. Further, plotting circles on a map is made more difficult by geometric distortions related to the map projection (chapter 13).

Since a navigator always has an estimate of his position, it is not necessary to plot the whole circles of equal altitude but rather their parts near the expected position. In the 19th century, two ingenious navigators developed ways to construct straight lines (secants and tangents of the circles of equal altitude) whose point of intersection approximates our position. These revolutionary methods, which marked the beginning of modern celestial navigation, will be explained later. In summary, finding one's position by astronomical observations includes three basic steps:

1. Measuring the altitudes or zenith distances of two or more chosen bodies (chapter 2).

2. Finding the geographic position of each body at the time of its observation (chapter 3).

3. Deriving the position from the above data (chapter 4&5).