Chapter 3

The Geographic Position (GP) of a Celestial Body

Geographic terms

In celestial navigation, the earth is regarded as a sphere. Although this is only an approximation, the geometry of the sphere is applied successfully, and the errors caused by the oblateness of the earth are usually negligible (see chapter 9).

Any circle on the surface of the earth whose plane passes through the center of the earth is called a **great circle**. Thus, a great circle is a circle with the greatest possible diameter on the surface of the earth. Any circle on the surface of the earth whose plane does not pass through the earth's center is called a **small circle**. The **equator** is the (only) great circle whose plane is perpendicular to the **polar axis**, the axis of rotation. Further, the equator is the only **parallel of latitude** being a great circle. Any other parallel of latitude is a small circle whose plane is parallel to the plane of the equator.
A **meridian** is a great circle going through the **geographic poles**, the points where the polar axis intersects the earth's surface. The **upper branch** of a meridian is the half from pole to pole passing through a given point, the **lower branch** is the opposite half. The **Greenwich meridian**, the meridian passing through the center of the transit instrument at the **Royal Greenwich Observatory**, was adopted as the **prime meridian** at the International Meridian Conference in October 1884. Its upper branch (0°) is the reference for measuring longitudes, its lower branch (180°) is known as the **International Dateline** (*Fig. 3-1*).

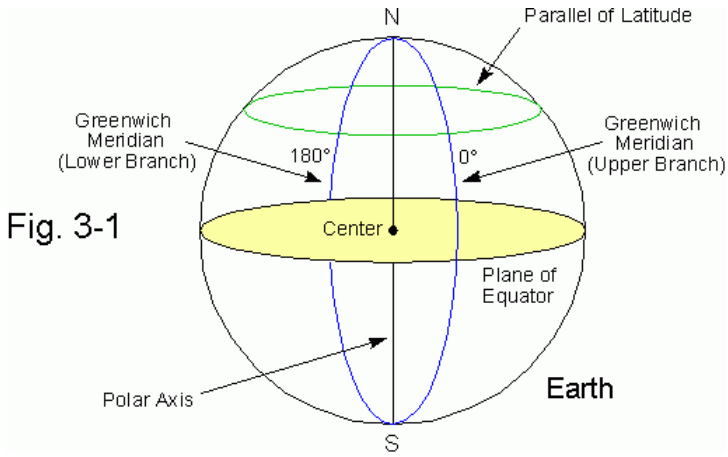


Fig. 3-1

Angles defining the position of a celestial body

The geographic position of a celestial body, GP, is defined by the **equatorial system of coordinates** (*Fig. 3-2*). The **Greenwich hour angle**, **GHA**, is the angular distance of GP **westward** from the upper branch of the Greenwich meridian (0°), measured from 0° through 360°. The **declination**, **Dec**, is the angular distance of GP from the plane of the equator, measured northward through +90° or southward through -90°. GHA and Dec are **geocentric coordinates** (measured at the center of the earth). The great circle going through the poles and GP is called **hour circle** (*Fig. 3-2*).

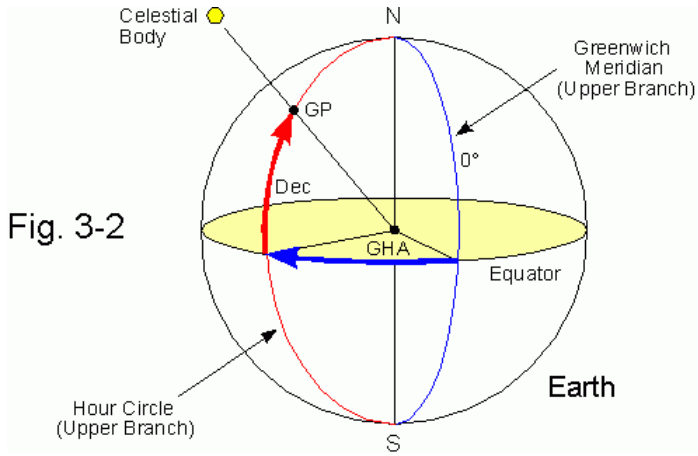


Fig. 3-2

GHA and Dec are equivalent to geocentric longitude and latitude with the exception that the longitude is measured from -(W)180° through +(E)180°.

Since the Greenwich meridian rotates with the earth from west to east, whereas each hour circle remains linked with the almost stationary position of the respective body in the sky, the GHA's of all celestial bodies increase as time progresses (approx. 15° per hour). In contrast to stars, the GHA's of sun, moon, and planets increase at slightly different (and variable) rates. This is attributable to the revolution of the planets (including the earth) around the sun and to the revolution of the moon around the earth, resulting in additional apparent motions of these bodies in the sky.

It is sometimes useful to measure the angular distance between the hour circle of a celestial body and the hour circle of a reference point in the sky instead of the Greenwich meridian because the angle thus obtained is independent of the earth's rotation. The angular distance of a body **westward** from the hour circle (upper branch) of the **first point of Aries**, measured from 0° through 360° is called **siderial hour angle**, **SHA**. The first point of Aries is the fictitious point in the sky where the sun passes through the plane of the earth's equator in spring (vernal point). The GHA of a body is the sum of the SHA of the body and the GHA of the first point of Aries, $\mathbf{GHA_{Aries}}$:

$$GHA = SHA + GHA_{Aries}$$

(If the resulting GHA is greater than 360°, subtract 360°.)

GHA_{Aries}, measured in time units (0-24h) instead of degrees, is called **Greenwich Siderial Time**, **GST**:

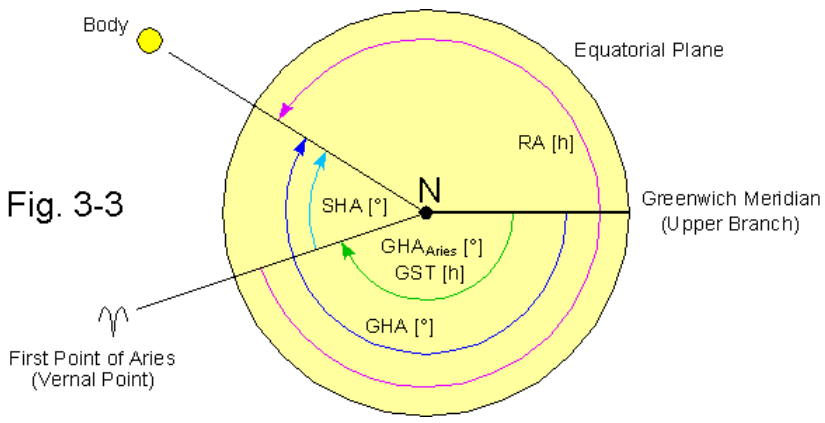
$$GST\,[h] = \frac{GHA_{Aries}\,[°]}{15} \quad \Leftrightarrow \quad GHA_{Aries}\,[°] = 15 \cdot GST\,[h]$$

The angular distance of a body measured in time units (0-24h) **eastward** from the hour circle of the first point of Aries is called **right ascension**, **RA**:

$$RA\,[h] = 24 - \frac{SHA\,[°]}{15} \quad \Leftrightarrow \quad SHA\,[°] = 360 - 15 \cdot RA\,[h]$$

Fig. 3-3 illustrates how the various hour angles are interrelated.

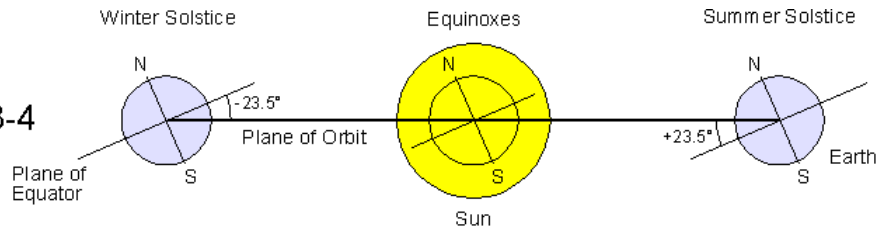
Fig. 3-3



Declinations are not affected by the rotation of the earth. The declinations of sun and planets change primarily due to the **obliquity of the ecliptic**, the inclination of the earth's equator to the plane of the earth's orbit (ecliptic). The declination of the sun, for example, varies periodically between ca. +23.5° at the time of the summer solstice and ca. -23.5° at the time of the winter solstice. At two moments during the course of a year the plane of the earth's equator passes through the center of the sun. Accordingly, the sun's declination passes through 0° (*Fig.3-4*).

When the sun is on the equator, day and night are equally long at any place on the earth. Therefore, these events are called **equinoxes** (equal nights). The apparent geocentric position of the sun in the sky at the instant of the vernal (spring) equinox marks the first point of Aries, the reference point for measuring siderial hour angles (see above).

Fig. 3-4



In addition, the declinations of the planets and the moon are influenced by the inclinations of their own orbits to the ecliptic. The plane of the moon's orbit, for example, is inclined to the ecliptic by approx. 5° and makes a tumbling movement (precession, see below) with a cycle time of 18.6 years (Saros cycle). As a result, the declination of the moon varies between approx. -28.5° and +28.5° at the beginning and at the end of the Saros cycle, and between approx. -18.5° and +18.5° in the middle of the Saros cycle.

Further, siderial hour angles and declinations of all bodies change slowly due to the influence of the **precession** of the earth's polar axis. Precession is a slow, circular movement of the polar axis along the surface of an imaginary double cone. One revolution takes about 26000 years (Platonic year). Thus, the vernal point moves along the equator at a rate of approx. 50" per year. In addition, the polar axis makes a nodding movement, called **nutation**, which causes small periodic fluctuations of the SHA's and declinations of all bodies. Last but not least, even stars are not fixed in space but have their own movements, contributing to a slow drift of their celestial coordinates.

The accurate prediction of geographic positions of celestial bodies requires complicated algorithms. Formulas for the calculation of low-precision **ephemerides** of the sun (accurate enough for celestial navigation) are given in chapter 15.

Time Measurement

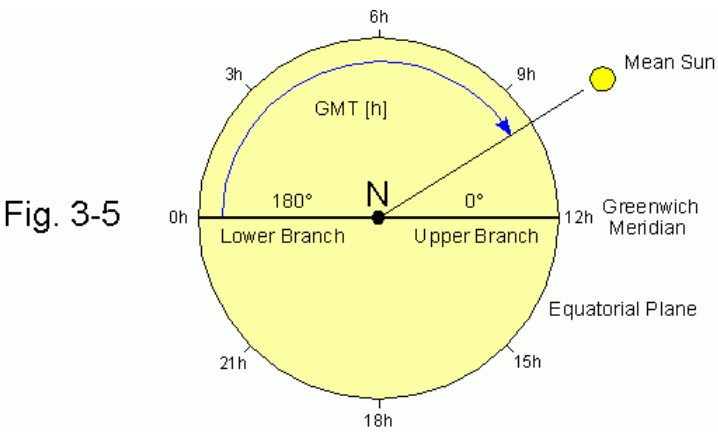
The time standard for celestial navigation is **Greenwich Mean Time**, **GMT** (now called **Universal Time**, **UT**). GMT is based upon the GHA of the (fictitious) **mean sun**:

$$GMT\,[h] = \frac{GHA_{MeanSun}\,[°]}{15} + 12$$

(If GMT is greater than 24 h, subtract 12 hours.)

In other words, GMT is the hour angle of the mean sun, expressed in hours, with respect to the lower branch of the Greenwich meridian (*Fig. 3-5*).

Fig. 3-5



By definition, the GHA of the mean sun increases by exactly 15° per hour, completing a 360° cycle in 24 hours. Celestial coordinates tabulated in the **Nautical Almanac** refer to GMT (UT).

The hourly increase of the GHA of the **apparent (observable) sun** is subject to periodic changes and is sometimes slightly greater, sometimes slightly smaller than 15° during the course of a year. This behavior is caused by the eccentricity of the earth's orbit and by the obliquity of the ecliptic. The time derived from the GHA of the apparent sun is called **Greenwich Apparent Time**, **GAT**. A sundial located at the Greenwich meridian, for example, would indicate GAT. The difference between GAT and GMT is called **equation of time, EoT**:

$$EoT = GAT - GMT$$

EoT varies periodically between approx. -16 minutes and +16 minutes. Predicted values for EoT for each day of the year (at 0:00 and 12:00 GMT) are given in the Nautical Almanac (grey background indicates negative EoT). EoT is needed when calculating times of sunrise and sunset, or determining a **noon longitude** (see chapter 6). Formulas for the calculation of EoT are given in chapter 15.

Due to the rapid change of GHA, celestial navigation requires accurate time measurement, and the time at the instant of observation should be noted to the second if possible. This is usually done by means of a chronometer and a stopwatch. The effects of time errors are dicussed in chapter 16. If GMT (UT) is not available, **UTC (Coordinated Universal Time)** can be used. UTC, based upon highly accurate atomic clocks, is the standard for radio time signals broadcast by, e. g., WWV or WWVH*. Since GMT (UT) is linked to the earth's rotating speed which decreases slowly and, moreover, with unpredictable irregularities, GMT (UT) and UTC tend to drift apart. For practical reasons, it is desirable to keep the difference between GMT (UT) and UTC sufficiently small. To ensure that the difference, DUT, never exceeds ±0.9 s, UTC is synchronized with UT by inserting or omitting leap seconds at certain times, if necessary. Current values for DUT are published by the **United States Naval Observatory**, **Earth Orientation Department**, on a regular basis (IERS Bulletin A).

$$UT = UTC + DUT$$

*It is most confusing that nowadays the term GMT is often used as a synonym for UTC instead of UT. GMT time signals from radio stations generally refer to UTC. In this publication, the term GMT is always used in the traditional (astronomical) sense, as explained above.

Terrestrial Dynamical Time, TDT, is an atomic time scale which is **not** synchronized with GMT (UT). It is a continuous and linear time measure used in astronomy (calculation of ephemerides) and space flight. TDT is presently (2001) approx. 1 minute ahead of GMT.

The Nautical Almanac

Predicted values for GHA and Dec of sun, moon and the navigational planets with reference to GMT (UT) are tabulated for each whole hour of the year on the daily pages of the **Nautical Almanac**, **N.A.**, and similar publications [12, 13]. GHA_{Aries} is tabulated in the same manner.

Listing GHA and Dec of all 57 fixed stars used in navigation for each whole hour of the year would require too much space. Since declinations of stars and (apparent) positions of stars relative to each other change only slowly, tabulated average siderial hour angles and declinations of stars for periods of 3 days are accurate enough for navigational applications.

GHA and Dec for each second of the year are obtained using the **interpolation tables** at the end of the N.A. (printed on tinted paper), as explained in the following directions:

1.

We note the exact time of observation (UTC or, preferably, UT), determined with a chronometer, for each celestial body.

2.

We look up the day of observation in the N.A. (two pages cover a period of three days).

3.

We go to the nearest whole hour preceding the time of observation and note GHA and Dec of the observed body. In case of a fixed star, we form the sum of GHA Aries and the SHA of the star, and note the average Dec. When observing sun or planets, we note the ***v*** and ***d*** factors given at the bottom of the appropriate column. For the moon, we take *v* and *d* for the nearest whole hour preceding the time of observation.

The quantity *v* is necessary to apply an additional correction to the following interpolation of the GHA of moon and planets. It is not required for stars. The sun does not require a *v* factor since the correction has been incorporated in the tabulated values for the sun's GHA.

The quantity *d*, which is negligible for stars, is the change of Dec during the time interval between the nearest whole hour preceding the observation and the nearest whole hour following the observation. It is needed for the interpolation of Dec.

4.

We look up the minute of observation in the interpolation tables (1 page for each 2 minutes of the hour), go to the second of observation, and note the **increment** from the appropriate column.

We enter one of the three columns to the right of the increment columns with the *v* and *d* factors and note the corresponding **corr**(ection) values (*v*-corr and *d*-corr).

The sign of *d*-corr depends on the trend of declination at the time of observation. It is positive if Dec at the whole hour following the observation is greater than Dec at the whole hour preceding the observation. Otherwise it is negative.

V-corr is negative for Venus and otherwise always positive.

5.

We form the sum of Dec and *d*-corr (if applicable).

We form the sum of GHA (or GHA Aries and SHA of star), increment, and *v*-corr (if applicable).

Interactive Computer Ephemeris

The **Interactive Computer Ephemeris**, **ICE**, developed by the U.S. Naval Observatory, is a DOS program (successor of the **Floppy Almanac**) for the calculation of ephemeral data for sun, moon, planets and stars.

ICE is FREEWARE (no longer supported by USNO), compact, easy to use, and provides a vast quantity of accurate astronomical data for a time span of almost 250 (!) years.

Among many other features, ICE calculates GHA and Dec for a given body and time as well as altitude and azimuth of the body for an **assumed position** (see chapter 4) and sextant altitude corrections. Since the calculated data are as accurate as those tabulated in the **Nautical Almanac** (approx. 0.1'), the program makes an adequate alternative, although a printed almanac (and sight reduction tables) should be kept as a backup in case of a computer failure.

The following instructions refer to the final version (0.51). Only program features relevant to navigation are explained.

1. Installation
Copy the program files to a chosen directory on the hard drive or to a floppy disk.

2. Getting Started
Change to the program directory (or floppy disk) and enter "ice". The main menu appears. Use the function keys F1 to F10 to navigate through the submenus. The program is more or less self-explanatory.

Go to the submenu INITIAL VALUES (F1). Follow the directions on the screen to enter date and time of observation (F1), assumed latitude (F2), assumed longitude (F3), and your local time zone (F6). Assumed latitude and longitude define your assumed position. Use the correct data format, as shown on the screen (decimal format for latitude and longitude). After entering the above data, press F7 to accept the values displayed.

To change the default values permanently, edit the file ice.dft with a text editor (after making a backup copy) and make the appropriate changes. Do not change the data format. The numbers have to be in columns 21-40.

An output file can be created to store calculated data. Go to the submenu FILE OUTPUT (F2) and enter a chosen file name, e.g., OUTPUT.TXT.

3. Calculation of Navigational Data
From the main menu, go to the submenu NAVIGATION (F7). Enter the name of the body. The program displays GHA and Dec of the body, GHA and Dec of the sun (if visible), and GHA of the vernal equinox for the time (UT) stored in INITIAL VALUES. Hc (computed altitude) and Zn (azimuth) mark the apparent position of the body as observed from the assumed position. Approximate altitude corrections (refraction, SD, PA), based upon Hc, are also displayed (for lower limb of body). The semidiameter of the moon includes augmentation. The coordinates calculated for Venus and Mars do **not** include phase correction. Therefore, the upper or lower limb (if visible) should be observed. ΔT is TDT-UT, the difference between **terrestrial dynamical time** and UT for the date given (presently approx. 1 min.).

Horizontal parallax and semidiameter of a body can be extracted indirectly, if required, from the submenu POSITIONS (F3). Choose APPARENT GEOCENTRIC POSITIONS (F1) and enter the name of the body (sun, moon, planets). The last column shows the distance of the center of the body from the center of the earth, measured in astronomical units (1 AU = $149.6 \cdot 10^6$ km). HP and SD are calculated as follows:

$$HP = \arcsin \frac{r_E\,[km]}{distance\,[km]} \qquad SD = \arcsin \frac{r_B\,[km]}{distance\,[km]}$$

r_E is the equatorial radius of the earth (6378 km). r_B is the radius of the body (Sun: 696260 km, Moon: 1378 km, Venus: 6052 km, Mars: 3397 km, Jupiter: 71398 km, Saturn: 60268 km).

The apparent geocentric positions refer to TDT, but the difference between TDT and UT has no significant effect on HP and SD.

To calculate times of rising and setting of a body, go to the submenu RISE & SET TIMES (F6) and enter the name of the body. The columns on the right display the time of rising, meridian transit, and setting for your assumed location (UT+xh, according to the time zone specified).

Multiyear Interactive Computer Almanac

The **Multiyear Interactive Computer Almanac**, **MICA**, is the successor of ICE. MICA 1.5 includes the time span from 1990 through 2005. Versions for DOS and Macintosh are on one CD-ROM. MICA provides highly accurate ephemerides primarily for astronomical applications.

For navigational purposes, zenith distance and azimuth of a body with respect to an assumed position can also be calculated.

MICA computes RA and Dec but not GHA. Since MICA calculates GST, GHA can be obtained by applying the formulas shown at the beginning of the chapter. The following instructions refer to the DOS version.

Right ascension and declination of a body can be accessed through the following menus and submenus:
Calculate
Positions
Objects (choose body)
Apparent
Geocentric
Equator of Date

Greenwich siderial time is accessed through:
Calculate
Time & Orientation
Siderial Time
(App.)

The knowledge of corrected altitude and geographic position of a body enables the navigator to establish a **line of position**, as will be explained in chapter 4.