Preizkus znanja v 3d iz eksponentne funkcije in iz logaritmov, 31.1. 2012
(vse naloge so enakovredne)

1. Rešite enačbo

$4^{x-1}+2\left(\frac{1}{16}\right)^{1-\frac{x}{2}}-3\cdot 0{,}25^{-x}+21=0$

$4^{x-1}+2\left(\frac{1}{16}\right)^{1-\frac{x}{2}}-3\cdot 0{,}25^{-x}+21=0 \Rightarrow 4^{x-1}+2\left(4^{-2}\right)^{1-\frac{x}{2}}-3\cdot\left(4^{-1}\right)^{-x}=-21 \Rightarrow$
$\Rightarrow 4^{x-1}+2\cdot 4^{-2+x}-3\cdot 4^{x}=-21 \Rightarrow 4^{x-2}\left(4+2-3\cdot 4^{2}\right)=-21 \Rightarrow 4^{x-2}\left(-42\right)=-21 \Rightarrow$
$\Rightarrow 4^{x-2}=2^{-1} \Rightarrow 2^{2x-4}=2^{-1} \Rightarrow 2x-4=-1 \Rightarrow \underline{\underline{x=\frac{3}{2}}}$

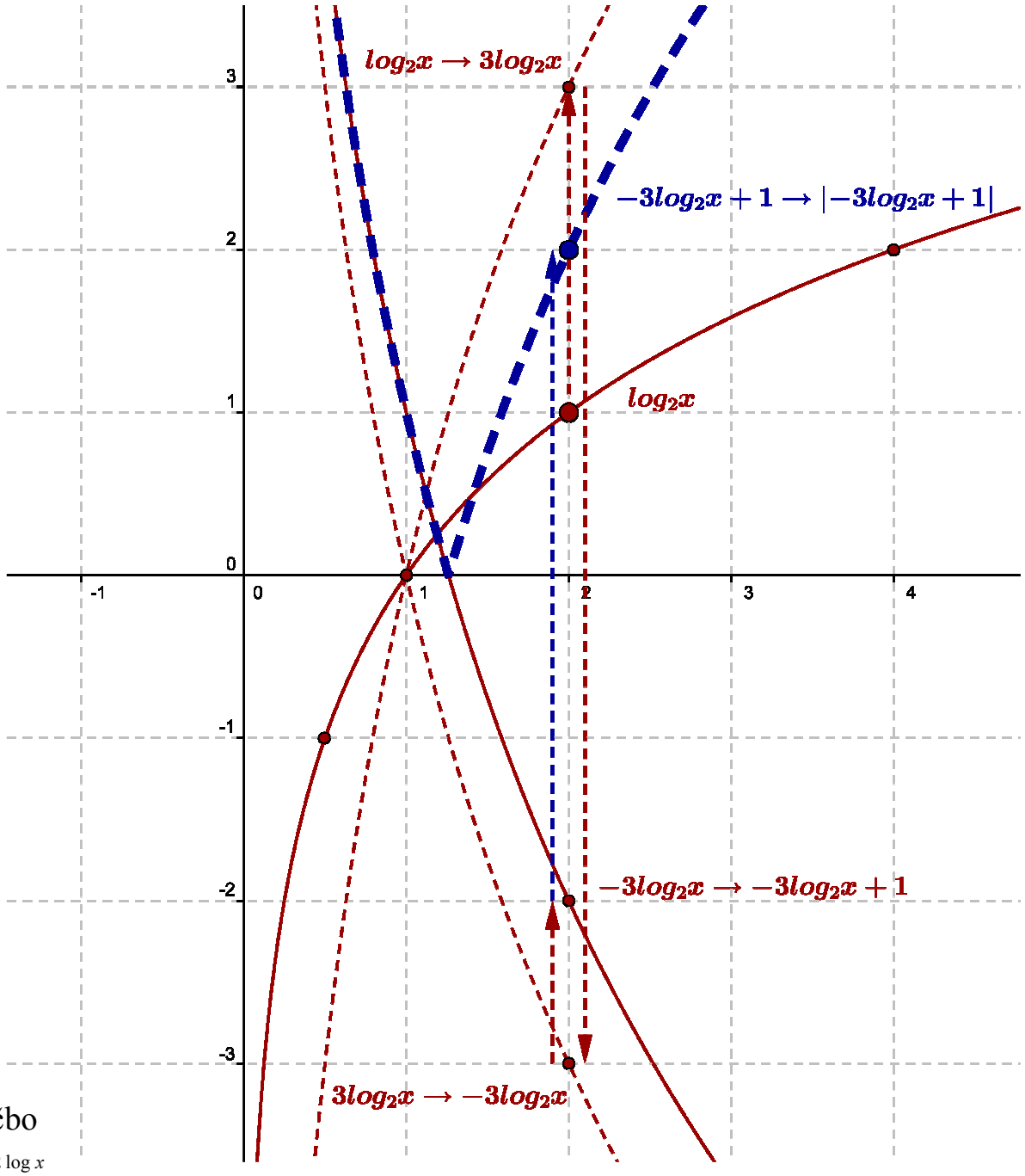
2. Določite x, če je $\log_2\left(6\log\left(9x+1\right)+\log_2 16\right)=4$.

$\log_2\left(6\log\left(9x+1\right)+\log_2 16\right)=4 \overset{\log_2 16=4}{\Rightarrow} 6\log\left(9x+1\right)+4=16 \Rightarrow \log\left(9x+1\right)=2 \overset{\log 100=2}{\Rightarrow}$
$\Rightarrow 9x+1=100 \Rightarrow \underline{\underline{x=11}}$
Preizkus: $L=\log_2\left(6\log 100+\log_2 16\right)=\log_2\left(6\cdot 2+4\right)=\log_2 16=4=D$

3. Narišite graf funkcije $f\left(x\right)=\left|1-3\log_2 x\right|$

Graf narišemo s transformacijami funkcije $\log_2 x$. Vsi grafi so rdeči, končni pa je moder.
$\log_2 x \xrightarrow[R_{3y}]{} 3\log_2 x \xrightarrow[Zx]{} -3\log_2 x \xrightarrow[P_{y+1}]{} -3\log_2 x+1 \xrightarrow[A_y]{} \left|-3\log_2 x+1\right|=\left|1-3\log_2 x\right|$
Še ničla $\left|1-3\log_2 x\right|=0 \Rightarrow 1-3\log_2 x=0 \Rightarrow 3\log_2 x=1 \Rightarrow \log_2 x=\frac{1}{3} \Rightarrow$
$\Rightarrow \underline{x=2^{\frac{1}{3}}=\sqrt[3]{2}\approx 1{,}26}$

GRAFI SO NA DRUGI STRANI.



4. Rešite enačbo

$10x^2 = x^{3+2\log x}$

$10x^2 = x^{3+2\log x} \,/_{\log} \Rightarrow \log(10x^2) = \log x^{3+2\log x} \Rightarrow \log 10 + \log x^2 = (3+2\log x)\log x \Rightarrow$

$\Rightarrow 1+2\log x = 3\log x + 2\log^2 x \overset{\log x = y}{\Rightarrow} 1+2y = 3y+2y^2 \Rightarrow 2y^2+y-1=0$

$y_{1,2} = \frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{4} \Rightarrow y_1 = \frac{1}{2} \text{ in } y_2 = -1$

$y_1 = \frac{1}{2} \Rightarrow \log x = \frac{1}{2} \Rightarrow \underline{\underline{x = \sqrt{10}}} \quad P: L = 10\left(\sqrt{10}\right)^2 = 100,\ D = \left(\sqrt{10}\right)^{3+2\log\sqrt{10}} = \left(\sqrt{10}\right)^4 = 100$

$y_2 = -1 \Rightarrow \log x = -1 \Rightarrow \underline{\underline{x = \frac{1}{10}}} \quad P: L = 10\left(\frac{1}{10}\right)^2 = \frac{1}{10},\ D = \left(\frac{1}{10}\right)^{3+2\log\frac{1}{10}} = \left(\frac{1}{10}\right)^{3-2} = \frac{1}{10}$

5. Rešite enačbo

$$\log_2(x-2) = \log_4(x^3-80) - \log_4 x$$

$$\log_2(x-2) = \log_4(x^3-80) - \log_4 x \overset{\text{prehod na novo osnovo}}{\Rightarrow} \log_2(x-2) = \frac{\log_2(x^3-80)}{\log_2 4} - \frac{\log_2 x}{\log_2 4} \Rightarrow$$

$$\Rightarrow \log_2(x-2) = \frac{\log_2(x^3-80)}{2} - \frac{\log_2 x}{2} \Rightarrow 2\log_2(x-2) = \log_2(x^3-80) - \log_2 x \Rightarrow$$

$$\Rightarrow \log_2(x-2)^2 = \log_2 \frac{x^3-80}{x} \Rightarrow x^2 - 4x + 4 = \frac{x^3-80}{x} \Rightarrow x^3 - 4x^2 + 4x = x^3 - 80 \Rightarrow$$

$$\Rightarrow 4x^2 - 4x - 80 = 0 \Rightarrow x^2 - x - 20 = 0 \Rightarrow (x-5)(x+4) = 0$$

$x_1 = 5 \Rightarrow \quad P: L = \log_2 3,\ D = \log_4(125-80) - \log_4 5 = \log_4(45:5) = \log_4 3^2 = \log_2 3$

$x_2 = -4 \Rightarrow \quad P: L = \log(-6)_{\text{ni definirano}}$

6. Leta 1993 je imela Slovenija 1,963 milijona prebivalcev, leta 2003 pa 1,936 milijona prebivalcev. Ob predpostavki, da bi se nadaljevala enaka stopnja rasti (upadanja), izračunajte, katerega leta bi imela Slovenija le še 1,500 milijona prebivalcev?

Prebivalstvo upada eksponentno, po geometrijskem zaporedju.

V 10 letih (od 1993 do 2003) je upadlo za faktor $\frac{1,936}{1,963}$, v enem letu pa

za faktor $\frac{1,936^{\frac{1}{10}}}{1,963} = \sqrt[10]{\frac{1,936}{1,963}} = 0,998616.$

V t letih (od leta 1993) bo upadlo od 1,963 na 1,500 $\Rightarrow 1,500 = 1,963 \cdot 0,998616^t$

Enačbo rešimo z logaritmiranjem $\log 1,500 = \log 1,963 + t \log 0,998616 \Rightarrow$

$$\Rightarrow t = \frac{\log 1,500 - \log 1,963}{\log 0,998616} = 194,236 \approx 194 \rightarrow \text{To se bo zgodilo leta } 1993 + 194 = 2187$$