Preizkus znanja v 2e iz korenov višjih stopenj, iracionalnih enačb, potenc z racionalnimi eksponenti in uporabe dopolnjevanja do popolnega kvadrata

26.11.2012 (vse naloge so enakovredne - za zadostno 50%)

1. Izračunajte, koliko je x:

$$\text{a)}\ \frac{\left(\sqrt[3]{2}\right)^6 \cdot 2^{-6}}{0{,}5^2 \cdot 8^{-\frac{2}{3}}} = x \cdot \frac{1}{\left(\sqrt[3]{2}\right)^9}$$

Poenostavite:

$$\text{b)}\ \sqrt[3]{\frac{x^4 y^3 \cdot \sqrt{\frac{64x^3}{9y^2}}}{9y^{-1}\sqrt{x^5}}}$$

Rešitev

$$\text{a)}\ \frac{\left(\sqrt[3]{2}\right)^6 \cdot 2^{-6}}{0{,}5^2 \cdot 8^{-\frac{2}{3}}} = x \cdot \frac{1}{\left(\sqrt[3]{2}\right)^9} \Rightarrow \frac{2^{\frac{6}{3}} \cdot 2^{-6}}{2^{-1\cdot 2} \cdot 2^{3\cdot\left(-\frac{2}{3}\right)}} = x \cdot \frac{1}{2^{\frac{9}{3}}} \Rightarrow \frac{2^2 \cdot 2^{-6}}{2^{-2} \cdot 2^{-2}} = x \cdot \frac{1}{2^3} \Rightarrow$$

$$\Rightarrow 2^{2-6+2+2} = x \cdot 2^{-3} \Rightarrow 1 = x \cdot 2^{-3} \Rightarrow \underline{\underline{x = 8}}$$

$$\text{b)}\ \sqrt[3]{\frac{x^4 y^3 \cdot \sqrt{\frac{64x^3}{9y^2}}}{9y^{-1}\sqrt{x^5}}} = \sqrt[3]{\frac{x^4 y^3 \cdot \frac{8x\sqrt{x}}{3y}}{9y^{-1}\, x^2\sqrt{x}}} = \sqrt[3]{\frac{8x^5 y^2 \sqrt{x}}{27y^{-1}x^2\sqrt{x}}} = \sqrt[3]{\frac{8x^3y^3}{27}} = \underline{\underline{\tfrac{2}{3}xy}}$$

2. Rešite iracionalno enačbo $\sqrt[3]{4x - 2\sqrt{x-2}} = 4.$

Rešitev $\sqrt[3]{4x - 2\sqrt{x-2}} = 4 \big/^{3} \Rightarrow 4x - 2\sqrt{x-2} = 64 \big/ {:2} \Rightarrow 2x - 32 = \sqrt{x-2} \big/^{2} \Rightarrow$

$$\Rightarrow 4x^2 - 128x + 1024 = x - 2 \Rightarrow 4x^2 - 129x + 1026 = 0 \big/ {:4} \Rightarrow x^2 - \tfrac{129}{4}x + \tfrac{1026}{4} = 0 \Rightarrow$$

$$\Rightarrow x^2 - \tfrac{129}{4}x + \tfrac{16641}{64} - \tfrac{16641}{64} + \tfrac{1026}{4} = 0 \Rightarrow \left(x - \tfrac{129}{8}\right)^2 - \tfrac{16641}{64} + \tfrac{16416}{64} = 0 \Rightarrow \left(x - \tfrac{129}{8}\right)^2 - \tfrac{225}{64} = 0 \Rightarrow$$

$$\Rightarrow \left(x - \tfrac{129}{8}\right)^2 - \left(\tfrac{15}{8}\right)^2 = 0 \Rightarrow \left(x - \tfrac{129}{8} - \tfrac{15}{8}\right)\left(x - \tfrac{129}{8} + \tfrac{15}{8}\right) = 0 \Rightarrow \left(x - \tfrac{144}{8}\right)\left(x - \tfrac{114}{8}\right) = 0 \Rightarrow$$

$$\Rightarrow (x - 18)\left(x - \tfrac{57}{4}\right) = 0$$

$$\underline{\underline{x_1 = 18}} \quad L = \sqrt[3]{4 \cdot 18 - 2\sqrt{16}} = \sqrt[3]{72 - 8} = 4 = D$$

$$x_2 = \tfrac{57}{4} \quad L = \sqrt[3]{4 \cdot \tfrac{57}{4} - 2\sqrt{\tfrac{57}{4} - 2}} = \sqrt[3]{57 - 2\sqrt{\tfrac{49}{4}}} = \sqrt[3]{57 - 7} = \sqrt[3]{50} \neq D$$

3. Z uvedbo nove neznanke $t=\sqrt[6]{x}$ rešite enačbo $\frac{8-\sqrt[3]{x}}{2\sqrt[6]{x}}=1$.

Rešitev $t=\sqrt[6]{x}\Rightarrow \frac{8-\sqrt[3]{x}}{2\sqrt[6]{x}}=1\Rightarrow \frac{8-\left(\sqrt[6]{x}\right)^2}{2\sqrt[6]{x}}=1\Rightarrow \frac{8-t^2}{2t}=1\Big/\cdot 2t\Rightarrow$

$\Rightarrow 8-t^2=2t\Rightarrow t^2+2t-8=0\Rightarrow (t+4)(t-2)=0$

$t_1=-4\Rightarrow \sqrt[6]{x}=-4_{\text{ni možno}}$

$t_2=2\Rightarrow \sqrt[6]{x}=2\Rightarrow \underline{\underline{x=2^6=64}}\quad L=\frac{8-\sqrt[3]{64}}{2\sqrt[6]{64}}=\frac{8-4}{2\cdot 2}=1=D$

4. Izračunajte:

a) $\sqrt{4\cdot\left(\frac{2}{9}\right)^{-2}-5\cdot 8^{\frac{4}{3}}}+5^{\frac{4}{5}}\cdot 5^{\frac{6}{5}}$ b) $\left(a^{-1}b^{\frac{1}{4}}\right)^{-2}\cdot\left(a^{-\frac{1}{2}}b^{\frac{1}{2}}\right)^4:\left(a^{\frac{1}{6}}b^{-\frac{5}{3}}\right)^{\frac{3}{2}}$

Rešitev

a) $\sqrt{4\cdot\left(\frac{2}{9}\right)^{-2}-5\cdot 8^{\frac{4}{3}}}+5^{\frac{4}{5}}\cdot 5^{\frac{6}{5}}=\sqrt{4\cdot\left(\frac{9}{2}\right)^{2}-5\cdot 2^{3\cdot\frac{4}{3}}}+5^{\frac{4}{5}+\frac{6}{5}}=\sqrt{81-5\cdot 2^4}+5^2=$

$=\sqrt{1}+25=\underline{\underline{26}}$

b) $\left(a^{-1}b^{\frac{1}{4}}\right)^{-2}\cdot\left(a^{-\frac{1}{2}}b^{\frac{1}{2}}\right)^4:\left(a^{\frac{1}{6}}b^{-\frac{5}{3}}\right)^{\frac{3}{2}}=a^{-1\cdot(-2)}b^{\frac{1}{4}\cdot(-2)}\cdot a^{-\frac{1}{2}\cdot 4}b^{\frac{1}{2}\cdot 4}:\left(a^{\frac{1}{6}\cdot\frac{3}{2}}b^{-\frac{5}{3}\cdot\frac{3}{2}}\right)=$

$=a^2b^{-\frac{1}{2}}\cdot a^{-2}b^2:\left(a^{\frac{1}{4}}b^{-\frac{5}{2}}\right)=a^{2+(-2)-\frac{1}{4}}b^{-\frac{1}{2}+2-\left(-\frac{5}{2}\right)}=a^{-\frac{1}{4}}b^{-\frac{1}{2}+\frac{4}{2}+\frac{5}{2}}=a^{-\frac{1}{4}}b^4=\frac{b^4\Big/\cdot\sqrt[4]{a^3}}{\sqrt[4]{a}\Big/\cdot\sqrt[4]{a^3}}=\underline{\underline{\frac{b^4}{a}\sqrt[4]{a^3}}}$

5. Z dopolnjevanjem do popolnega kvadrata rešite enačbo$_{\text{(lahko preverite po Vietovem pravilu)}}$

$3x^2+7x+2=0$

Rešitev $3x^2+7x+2=0\Big/:3\Rightarrow x^2+\frac{7}{3}x+\frac{2}{3}=0\Rightarrow x^2+\frac{7}{3}x+\frac{49}{36}-\frac{49}{36}+\frac{2}{3}=0\Rightarrow$

$\Rightarrow\left(x+\frac{7}{6}\right)^2-\frac{49}{36}+\frac{24}{36}=0\Rightarrow\left(x+\frac{7}{6}\right)^2=\frac{25}{36}\Rightarrow x+\frac{7}{6}=\pm\frac{5}{6}\Rightarrow$

$\underline{\underline{x_1=-\frac{7}{6}+\frac{5}{6}=-\frac{1}{3}\text{ in }x_2=-\frac{7}{6}-\frac{5}{6}=-2}}$

Vietovo pravilo $3\cdot 2=6=1\cdot 6$ ker $1+6=7$

$3x^2+7x+2=0\Rightarrow 3x^2+x+6x+2=0\Rightarrow x(3x+1)+2(3x+1)=0\Rightarrow$

$\Rightarrow(3x+1)(x+2)=0\Rightarrow \underline{\underline{x_1=-\frac{1}{3}\text{ in }x_2=-2}}$

6. Razstavite kvadratni tročlenik z dopolnjevanjem do popolnega kvadrata

$x^2+\frac{5}{6}x-1$

Rešitev $x^2+\frac{5}{6}x-1=x^2+\frac{5}{6}x+\frac{25}{144}-\frac{25}{144}-\frac{144}{144}=\left(x+\frac{5}{12}\right)^2-\frac{25}{144}-\frac{144}{144}=$

$=\left(x+\frac{5}{12}\right)^2-\frac{169}{144}=\left(x+\frac{5}{12}\right)^2-\left(\frac{13}{12}\right)^2=\left(x+\frac{5}{12}-\frac{13}{12}\right)\left(x+\frac{5}{12}+\frac{13}{12}\right)=$

$=\underline{\underline{\left(x-\frac{2}{3}\right)\left(x+\frac{3}{2}\right)}}$