1a$_A$ Preizkus znanja: racionalna števila (ulomki, potence s celimi eksponenti, decimalni zapis ulomkov, linearne enačbe, sistemi linearnih enačb - dveh oz. treh), realna števila (kvadratni in kubični koren)   4.4.11

Kriterij: 50%-zad, 62,5%-db, 75%-pdb, 87,5%-odl

Dve podnalogi sta dodatno, zato je 50% že 15 točk.

3 točke **1a Izračunajte** $\left(\dfrac{a^2 - a - 6}{a^2 - 3a} + \dfrac{2 - a}{a + 1} \right) : \dfrac{10a + 4}{a^2 + 2a + 1}$

$$\left(\frac{a^2 - a - 6}{a^2 - 3a} + \frac{2 - a}{a + 1} \right) : \frac{10a + 4}{a^2 + 2a + 1} = \left(\frac{(a-3)(a+2)}{a(a-3)} + \frac{2 - a}{a + 1} \right) \cdot \frac{(a+1)^2}{2(5a + 2)} =$$

$$= \frac{(a+2)(a+1) + (2-a)a}{a(a+1)} \cdot \frac{(a+1)^2}{2(5a + 2)} = \frac{a^2 + 3a + 2 + 2a - a^2}{a(a+1)} \cdot \frac{(a+1)^2}{2(5a + 2)} =$$

$$= \frac{5a + 2}{a} \cdot \frac{a + 1}{2(5a + 2)} = \frac{a + 1}{2a}$$

3 točke **b Izračunajte** $\dfrac{\frac{1}{a+2}}{\frac{1}{2a} - \frac{1}{2a+4}}$

$$\frac{\frac{1}{a+2}}{\frac{1}{2a} - \frac{1}{2a+4}} = \frac{\frac{1}{a+2} \Big/ \cdot 2a(a+2)}{\frac{1}{2a} - \frac{1}{2(a+2)} \Big/ \cdot 2a(a+2)} = \frac{2a}{a + 2 - a} = \frac{2a}{2} = a$$

3 točke **2a Poenostavite izraz** $\left(a^{-2} b^2 \right)^3 \cdot \left((-a)^{-3} (-b)^3 \right)^{-2}$

$$\left(a^{-2} b^2 \right)^3 \cdot \left((-a)^{-3} (-b)^3 \right)^{-2} = a^{-6} b^6 \cdot (-a)^6 (-b)^{-6} = a^{-6} b^6 \cdot a^6 b^{-6} = a^0 b^0 = 1$$

3 točke **b Razstavite izraz** _{pišite kot ulomke} $5x^{-2} y^2 - 6x^{-1} y + 1$

$$5 \frac{y^2}{x^2} - 6 \frac{y}{x} + 1 = \frac{5y^2 - 6xy + x^2}{x^2} = \frac{x^2 - 6xy + 5y^2}{x^2} = \frac{(x - 5y)(x - y)}{x^2}$$

3 točke **c Okrajšajte ulomek** $\dfrac{5^x + 5^{x+1}}{5^{x-1} - 5^{x+1}}$

$$\frac{5^x + 5^{x+1}}{5^{x-1} - 5^{x+1}} = \frac{5^x(1 + 5)}{5^{x-1}(1 - 5^2)} = \frac{6}{5^{-1}(-24)} = \frac{6 \cdot 5}{-24} = -\frac{5}{4} = -1\frac{1}{4}$$

3 točke **3 Izračunajte z ulomki** $0{,}2 \cdot 0{,}8\overline{3} - 1{,}75 : \frac{3}{4}$

$\begin{aligned} x &= 0{,}8\overline{3} \\ 10x &= 8{,}\overline{3} \\ 100x &= 83{,}\overline{3} \\ \hline 90x &= 75 \end{aligned}$

$$\frac{2}{10} \cdot \frac{75}{90} - 1\frac{3}{4} \cdot \frac{4}{3} = \frac{1}{5} \cdot \frac{5}{6} - \frac{7}{4} \cdot \frac{4}{3} = \frac{1}{6} - \frac{7}{3} = \frac{1}{6} - \frac{14}{6} = -\frac{13}{6} = -2\frac{1}{6}$$

3 točke **4a Rešite enačbo** $\dfrac{3}{x + 2} - 2(x - 3)^{-1} = \dfrac{x - 1}{x^2 - x - 6}$

$$\frac{3}{x + 2} - \frac{2}{x - 3} = \frac{x - 1}{(x + 2)(x - 3)} \Big/ \cdot (x+2)(x-3) \Rightarrow 3x - 9 - 2x - 4 = x - 1$$

$$x - 13 = x - 1 \Rightarrow -13 = -1 \Rightarrow x \in \varnothing$$

3 točke **b Rešite razcepno enačbo** $(x + 2)^2 + 3(x + 2) + 2 = 0$

$$(x + 2)^2 + 3(x + 2) + 2 = 0 \Rightarrow (x + 2 + 2)(x + 2 + 1) = 0$$

$$(x + 4)(x + 3) = 0 \Rightarrow x_1 = -4, \ x_2 = -3$$

3 točke **5a Miha je bil pred 18 leti dvakrat starejši od Darka, čez tri leta pa bosta skupaj imela 60 let. Koliko let imata Miha in Darko?**

Nastavitev enačb:

$$m - 18 = 2(d - 18) \text{ in } (m + 3) + (d + 3) = 60 \Rightarrow m + d = 60 - 6 = 54$$

Primerjalni način - primerjamo m

$$2(d - 18) + 18 = 54 - d \Rightarrow 2d - 36 + 18 = 54 - d \Rightarrow 3d = 54 + 18 = 72$$

$d = 24$ let in $m = 54 - 24 = 30$ let

3 točke **b Izračunajte ulomka, ki ste ju sešteli $\dfrac{2 - 2a}{a^2 - 10a + 24}$**

$$\frac{2 - 2a}{a^2 - 10a + 24} = \frac{2 - 2a}{(a - 6)(a - 4)} = \frac{A}{a - 6} + \frac{B}{a - 4} = \frac{A(a - 4) + B(a - 6)}{(a - 6)(a - 4)} =$$

$$= \frac{Aa - 4A + Ba - 6B}{(a - 6)(a - 4)} = \frac{\overbrace{(A + B)a + (-4A - 6B)}^{= -2a + 2}}{(a - 6)(a - 4)} \xrightarrow{\text{dve enačbi}} \Rightarrow \begin{array}{l} A + B = -2 \,/ \cdot 2 \\ -4A - 6B = 2 \,/ \cdot 2 \end{array}$$

$$\begin{array}{l} 2A + 2B = -4 \\ -2A - 3B = 1 \end{array} \Rightarrow -B = -3 \Rightarrow B = 3 \text{ in } A = -2 - B = -2 - 3 = -5$$
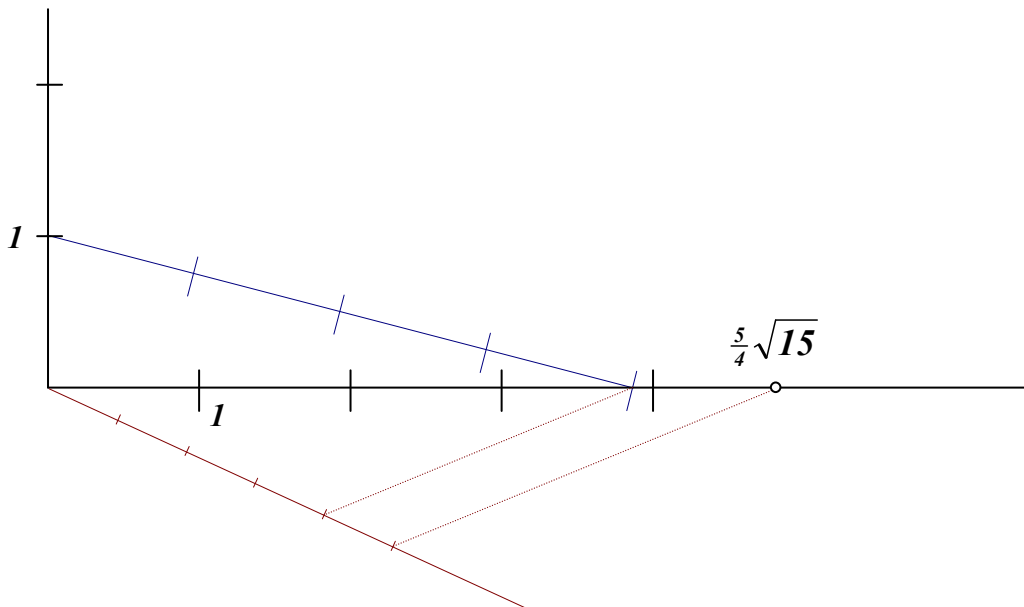
$$\frac{2 - 2a}{a^2 - 10a + 24} = -\frac{5}{a - 6} + \frac{3}{a - 4}$$

3 točke **4a Natančno izračunajte $\left(2\sqrt{3} - \sqrt{2}\right)^3$**

$$\left(2\sqrt{3} - \sqrt{2}\right)^3 = 8 \cdot 3\sqrt{3} - 3 \cdot 12\sqrt{2} + 3 \cdot 2\sqrt{3} \cdot 2 - 2\sqrt{2} =$$

$$= 24\sqrt{3} - 36\sqrt{2} + 12\sqrt{3} - 2\sqrt{2} = 36\sqrt{3} - 38\sqrt{2}$$

3 točke **b Na številski osi konstruirajte število $\frac{5}{4}\sqrt{15}$**

$4^2 - 1^2 = 15$ Pravokotni trikotnik s hipotenuzo 4 in eno kateto 1, ima drugo kateto dolgo $\sqrt{15}$.

1a_B Preizkus znanja: racionalna števila (ulomki, potence s celimi eksponenti, decimalni zapis ulomkov, linearne enačbe, sistemi linearnih enačb - dveh oz. treh), realna števila (kvadratni in kubični koren)   4.4.11
Kriterij: 50%-zad, 62,5%-db, 75%-pdb, 87,5%-odl

Dve podnalogi sta dodatno, zato je 50% že 15 točk.

3 točke **1a Izračunajte** $\left(\dfrac{x^2-3x-10}{x^2-5x}+\dfrac{2-x}{x+1}\right):\dfrac{10x+4}{x^2+2x+1}$

$$\left(\dfrac{x^2-3x-10}{x^2-5x}+\dfrac{2-x}{x+1}\right):\dfrac{10x+4}{x^2+2x+1}=\left(\dfrac{(x-5)(x+2)}{x(x-5)}+\dfrac{2-x}{x+1}\right)\cdot\dfrac{(x+1)^2}{2(5x+2)}=$$

$$=\dfrac{(x+2)(x+1)+(2-x)x}{x(x+1)}\cdot\dfrac{(x+1)^2}{2(5x+2)}=\dfrac{x^2+3x+2+2x-x^2}{x(x+1)}\cdot\dfrac{(x+1)^2}{2(5x+2)}=$$

$$=\dfrac{5x+2}{x}\cdot\dfrac{x+1}{2(5x+2)}=\dfrac{x+1}{2x}$$

3 točke **b Izračunajte** $\dfrac{\frac{1}{x+3}}{\frac{1}{3x}-\frac{1}{3x+9}}$

$$\dfrac{\frac{1}{x+3}}{\frac{1}{3x}-\frac{1}{3x+9}}=\dfrac{\frac{1}{x+3}\Big/\cdot 3x(x+3)}{\frac{1}{3x}-\frac{1}{3(x+3)}\Big/\cdot 3x(x+3)}=\dfrac{3x}{x+3-x}=\dfrac{3x}{3}=x$$

3 točke **2a Poenostavite izraz** $\left(x^2y^{-2}\right)^{-3}\cdot\left((-x)^3(-y)^{-3}\right)^2$

$$\left(x^2y^{-2}\right)^{-3}\cdot\left((-x)^3(-y)^{-3}\right)^2=x^{-6}y^6(-x)^6(-y)^{-6^2}=x^{-6}y^6x^6y^{-6}=x^0y^0=1$$

3 točke **b Razstavite izraz** *pišite kot ulomke* $4x^{-2}y^2-5x^{-1}y+1$

$$4\dfrac{y^2}{x^2}-5\dfrac{y}{x}+1=\dfrac{4y^2-5xy+x^2}{x^2}=\dfrac{x^2-5xy+4y^2}{x^2}=\dfrac{(x-4y)(x-y)}{x^2}$$

3 točke **c Okrajšajte ulomek** $\dfrac{3^x+2\cdot 3^{x+1}}{3^{x-1}-3^{x+1}}$

$$\dfrac{3^x+2\cdot 3^{x+1}}{3^{x-1}-3^{x+1}}=\dfrac{3^x(1+2\cdot 3)}{3^{x-1}(1-3^2)}=\dfrac{7}{3^{-1}(-8)}=-\dfrac{21}{8}=-2\dfrac{5}{8}$$

3 točke **3. Izračunajte z ulomki** $0{,}2\cdot 0{,}8\overline{3}-\frac{7}{4}:0{,}75$

$$\begin{array}{l}x=0{,}8\overline{3}\\10x=8{,}\overline{3}\\100x=83{,}\overline{3}\\\hline 90x=75\end{array}\quad \dfrac{2}{10}\cdot\dfrac{75}{90}-\dfrac{7}{4}:\dfrac{3}{4}=\dfrac{1}{5}\cdot\dfrac{5}{6}-\dfrac{7}{4}\cdot\dfrac{4}{3}=\dfrac{1}{6}-\dfrac{7}{3}=\dfrac{1}{6}-\dfrac{14}{6}=-\dfrac{13}{6}=-2\dfrac{1}{6}$$

3 točke **4. Rešite enačbo** $\dfrac{2}{x-3}-3(x+2)^{-1}=\dfrac{1-x}{x^2-x-6}$

$$\dfrac{2}{x-3}-\dfrac{3}{x+2}=\dfrac{1-x}{(x-3)(x+2)}\Big/\cdot(x-3)(x+2)\Rightarrow 2x+4-3x+9=1-x$$

$$-x+13=1-x\Rightarrow 13=1\Rightarrow x\in\varnothing$$

3 točke **b Rešite razcepno enačbo** $(x+3)^2+4(x+3)+3=0$

$$(x+3)^2+4(x+3)+3=0\Rightarrow(x+3+3)(x+3+1)=0$$

$$(x+6)(x+4)=0\Rightarrow x_1=-6,\ x_2=-4$$

3 točke **5a** *Miha je bil pred 21 leti trikrat starejši od Darka, čez tri leta*
pa bosta skupaj imela 60 let. Koliko let imata Miha in Darko?

Nastavitev enačb:

$$m - 21 = 3(d - 21) \text{ in } (m + 3) + (d + 3) = 60 \Rightarrow m + d = 60 - 6 = 54$$

Primerjalni način - primerjamo m

$$3(d - 21) + 21 = 54 - d \Rightarrow 3d - 63 + 21 = 54 - d \Rightarrow 4d = 54 + 42 = 96$$

$$d = 24 \text{ let in } m = 54 - 24 = 30 \text{ let}$$

3 točke **b** *Izračunajte ulomka, ki ste ju sešteli* $\dfrac{2a + 2}{a^2 - 10a + 24}$

$$\frac{2a + 2}{a^2 - 10a + 24} = \frac{2a + 2}{(a - 6)(a - 4)} = \frac{A}{a - 6} + \frac{B}{a - 4} = \frac{A(a - 4) + B(a - 6)}{(a - 6)(a - 4)} =$$

$$= \frac{Aa - 4A + Ba - 6B}{(a - 6)(a - 4)} = \frac{\overbrace{(A + B)a + (-4A - 6B)}^{=2a+2}}{(a - 6)(a - 4)} \xrightarrow{\text{dve enačbi}} \Rightarrow \begin{matrix} A + B = 2 /\cdot 2 \\ -4A - 6B = 2 /:2 \end{matrix}$$

$$\begin{matrix} 2A + 2B = 4 \\ -2A - 3B = 1 \end{matrix} \Rightarrow -B = 5 \Rightarrow B = -5 \text{ in } A = 2 - B = 2 + 5 = 7$$

$$\frac{2a + 2}{a^2 - 10a + 24} = \frac{7}{a - 6} - \frac{5}{a - 4}$$

3 točk **6.** *Natančno izračunajte :* $\left(2\sqrt{2} - \sqrt{3}\right)^3$

$$\left(2\sqrt{2} - \sqrt{3}\right)^3 = 8 \cdot 2\sqrt{2} - 3 \cdot 8\sqrt{3} + 3 \cdot 2\sqrt{2} \cdot 3 - 3\sqrt{3} =$$

$$= 16\sqrt{2} - 24\sqrt{3} + 18\sqrt{2} - 3\sqrt{3} = 34\sqrt{2} - 27\sqrt{3}$$

3 točke **b** *Na številski osi konstruirajte število* $\frac{5}{3}\sqrt{17}$

$4^2 + 1^2 = 17$ Pravokotni trikotnik z eno kateto 4 in drugo kateto 1,
ima hipotenuzo dolgo $\sqrt{17}$.