CAVITY THEORY

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Cavity Theory

Question:

Suppose we know the dose to the dosimeter, how can this be translated to the dose to the medium at the position of the dosimeter ?

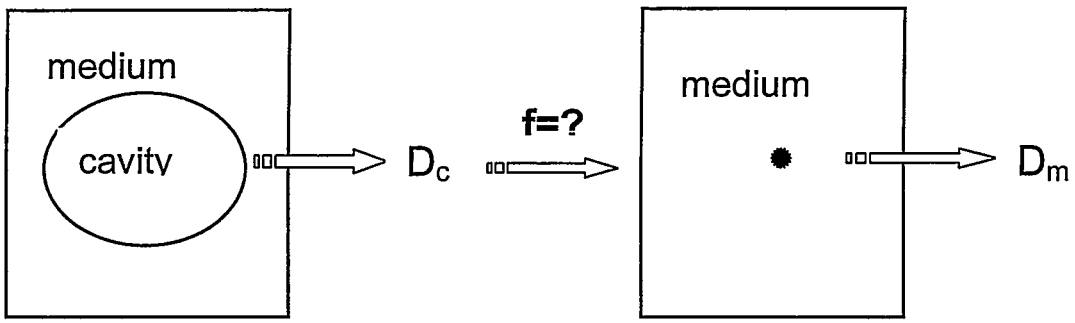
Answer:

$$D_m = f\, D_c$$

D_m the dose to the medium (m) at the position of the dosimeter.

D_c the dose to the dosimeter, which represents a cavity within the medium.

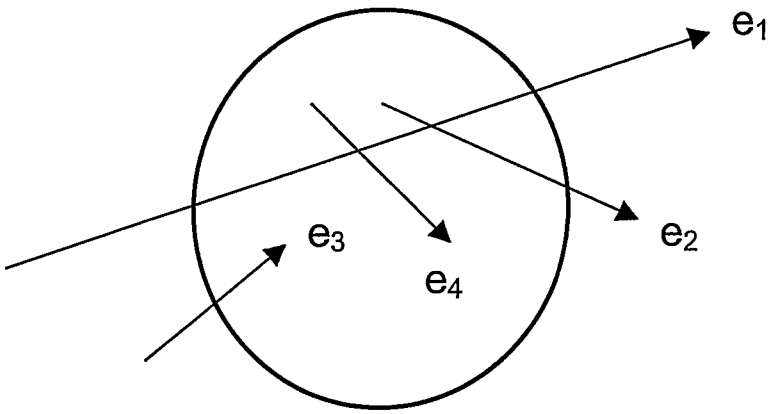
f cavity factor ($=D_m/D_c$) representing the dose to the medium per unit dose to the dosimeter.



Cavity theory is meant to determine the f-factor for a variety of conditions, namely, when the cavity is "small", "medium", or "large".

Cavity theory is not needed when, either the dosimeter and medium are of the same material ($c=m \Rightarrow D_c=D_m \Rightarrow f=1$), or the dosimeter is calibrated against the medium and is irradiated in the same field (the f-factor cancels out).

Terminology



e_1: charged particle that completely traverses the cavity: "crosser".

e_2: charged particle that originates in the cavity but stops outside: "starter".

e_3: charged particle that originated outside the cavity but stops inside: "stopper".

e_4: charged particle that starts and stops within the cavity: "insider".

Small Cavities: Bragg-Gray Theory

Assumptions:

1. The dose in the cavity is deposited entirely by crossers. That is, charged particles entering the cavity are assumed not to stop inside its volume and, also, in case the primary beam consists of un-charged particles, no secondaries are produced in the cavity.

2. The presence of the cavity does not perturb the charged particle field. $\Rightarrow \Phi_m = \Phi_c$

For monoenergetic beams:

$$\frac{D_m}{D_c} \overset{(1)}{=} \frac{\Phi_m}{\Phi_c}\frac{(S/\rho)_m}{(S/\rho)_c} \overset{(2)}{=} \frac{(S/\rho)_m}{(S/\rho)_c}$$

Tables

where S denotes the collision stopping-power ($=S_{col}$).

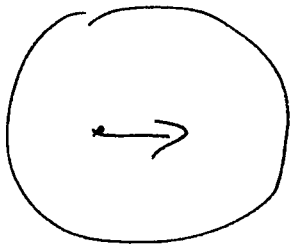
For a spectrum:

$$\frac{D_m}{D_c} \overset{(1)}{=} \frac{\int_0^{T_{max}} (d\Phi/dT)_m (S/\rho)_m \, dT}{\int_0^{T_{max}} (d\Phi/dT)_c (S/\rho)_c \, dT} = \frac{\Phi_m}{\Phi_c}\frac{\overline{(S/\rho)}_m}{\overline{(S/\rho)}_c} \overset{(2)}{=} \frac{\overline{(S/\rho)}_m}{\overline{(S/\rho)}_c}$$

Thus, the "small" cavity f-factor in the Bragg-Gray theory is:

$$f \overset{B-G}{=} \frac{\overline{(S/\rho)}_m}{\overline{(S/\rho)}_c}$$

Large Cavities: CPE



Assumptions:

1. CPE is satisfied at some depth within the cavity. That is, the dose in the cavity (at CPE depth) is entirely due to insiders produced from the primary un-charged particle beam.

2. The presence of the cavity does not perturb the primary un-charged particle field. $\Psi_m = \Psi_c$

For monoenergetic beams:

$$\frac{D_m}{D_c} \overset{(1)}{=} \frac{\Psi_m}{\Psi_c}\frac{(\mu_{en}/\rho)_m}{(\mu_{en}/\rho)_c} \overset{(2)}{=} \frac{(\mu_{en}/\rho)_m}{(\mu_{en}/\rho)_c}$$

For a spectrum:

$$\frac{D_m}{D_c} \overset{(1)}{=} \frac{\int_0^{E_{max}} (d\Psi/dE)_m (\mu_{en}/\rho)_m \, dE}{\int_0^{E_{max}} (d\Psi/dE)_c (\mu_{en}/\rho)_c \, dE} = \frac{\Psi_m}{\Psi_c}\frac{\overline{(\mu_{en}/\rho)_m}}{\overline{(\mu_{en}/\rho)_c}} \overset{(2)}{=} \frac{\overline{(\mu_{en}/\rho)_m}}{\overline{(\mu_{en}/\rho)_c}}$$

Thus, the "large" cavity f-factor under CPE is:

$$f \overset{CPE}{=} \frac{\overline{(\mu_{en}/\rho)_m}}{\overline{(\mu_{en}/\rho)_c}}$$

Medium Cavities: Burlin Theory

Assumptions:

1. The dose in the cavity is due to crossers, as well as, to insiders produced by the primary un-charged particle beam.

2. CPE exists at some depth inside the cavity.

3. The presence of the cavity does not perturb either the secondary charged particle field crossing it or the primary un-charged particle field interacting in it.

Combining the Bragg-Gray theory with CPE, the f-factor for medium cavities in the Burlin theory is:

$$f^{Burlin} = d\frac{\overline{(S/\rho)_m}}{\overline{(S/\rho)_c}} + (1-d)\frac{\overline{(\mu_{en}/\rho)_m}}{\overline{(\mu_{en}/\rho)_c}}$$

CROSSERS STOPPERS

where the S/ρ is averaged over the fluence spectrum of the charged particles crossing the cavity, whereas the μ/ρ is averaged over the energy fluence spectrum of the un-charged particles interacting within the cavity.

The parameter d depends on the size of the cavity with respect to the range of the charged particles striking it:

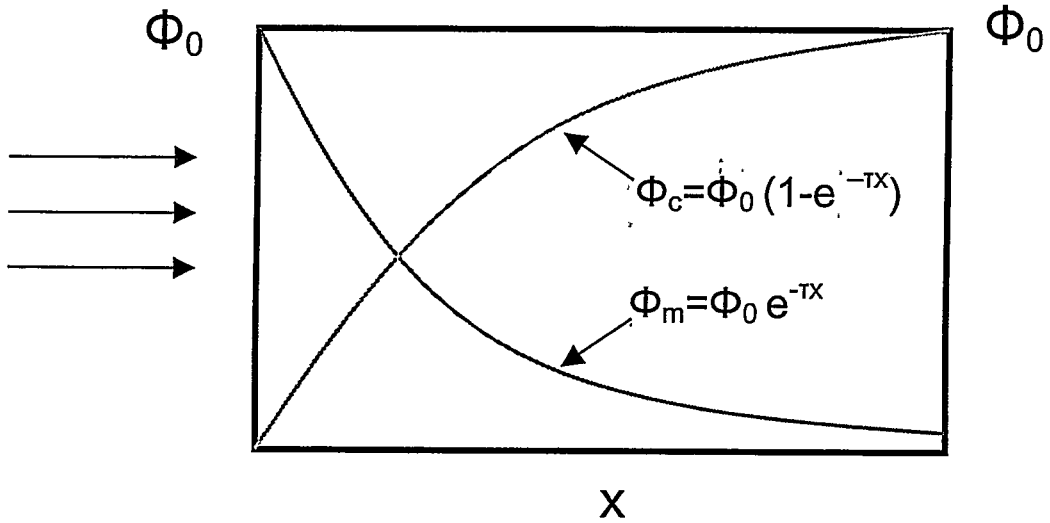
$$x << R \quad \Rightarrow \quad d \approx 1 \quad (B-G)$$

$$x >> R \quad \Rightarrow \quad d \approx 0 \quad (CPE)$$

For the general case where 0<d<1, the value of d is obtained by examining the relevant contribution to the cavity dose of the crossers versus the insiders. This is accomplished by two further assumptions:

4. The fluence of the crossers is attenuated exponentially, whereas the fluence of the insiders also builds up exponentially according to the same attenuation coefficient.

5. The equilibrium fluence for crossers and insiders is the same.



$$d \equiv \frac{\overline{\Phi}_m}{\Phi_0} = \frac{1}{\Phi_0} \frac{\int_0^J \Phi_0 e^{-\tau x} dx}{\int_0^J dx} = \frac{1 - e^{-\tau J}}{\tau J}$$

$$1 - d = \frac{\overline{\Phi}_c}{\Phi_0} = \frac{1}{\Phi_0} \frac{\int_0^J \Phi_0 (1 - e^{-\tau x}) dx}{\int_0^J dx} = \frac{\tau J + e^{-\tau J} - 1}{\tau J}$$

In Practice

Callibration: The dosimeter is irradiated in a known field, which delivers a known dose D_m to the medium of interest, and its response P is measured:

$$D_m = f\, D_c = f\,(kP)$$

where k the dose-response factor for the dosimeter for the calibration field.

Irradiation: The dosimeter is then irradiated in a different field, which delivers an un-known dose D_m' to the medium of interest, and its response P' is measured:

$$D_m' = f'\, D_c' = f'\,(k'P')$$

where k' the dose-response factor for the dosimeter for the irradiation field.

Calculation:

$$D_m' = D_m \frac{f'\, D_c'}{f\, D_c} = D_m \frac{f'}{f} \frac{(k'P')}{(kP)} \approx D_m \frac{f'}{f} \frac{P'}{P}$$

where f and f' are the cavity factors for the particular dosimeter and medium at the calibration and irradiation energies, respectively.