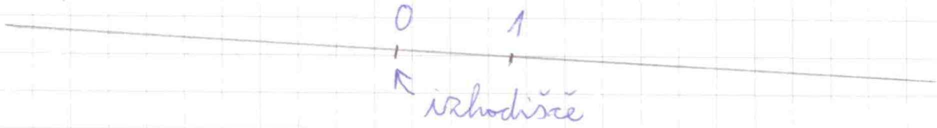
$\mathbb{R}^3$ realen trirazsežni prostor

vidik oz. pristop: analitičen (algebrajski), geometrijski

$\mathbb{R}$ model: premica opremljena s koordinatnim sistemom (+ dve točki) izhodišče in enota



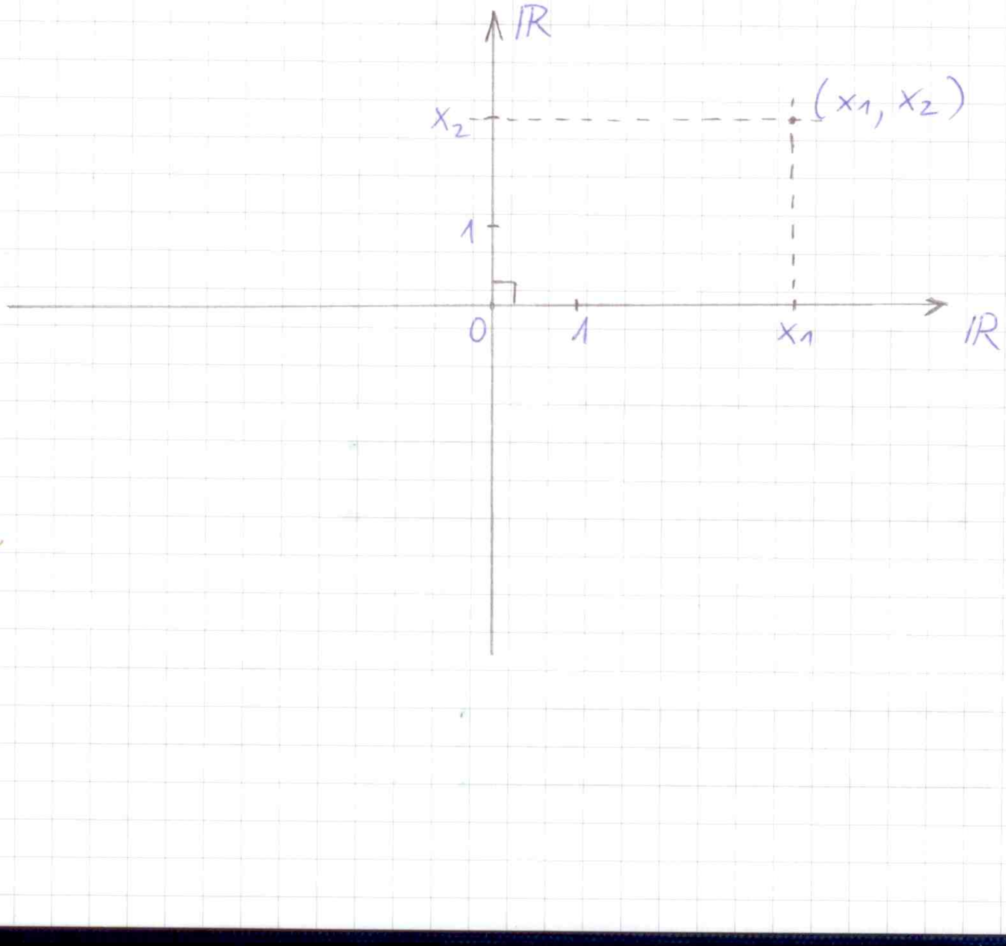
$\mathbb{R}^2$ dvorazsežni vektorski prostor

evklidska ravnina, kartezični produkt

$$\mathbb{R}^2 \equiv \mathbb{R} \times \mathbb{R} = \{(x_1, x_2) : x_1 \in \mathbb{R},\ x_2 \in \mathbb{R}\}$$

komponenti urejenega para, koordinati točke

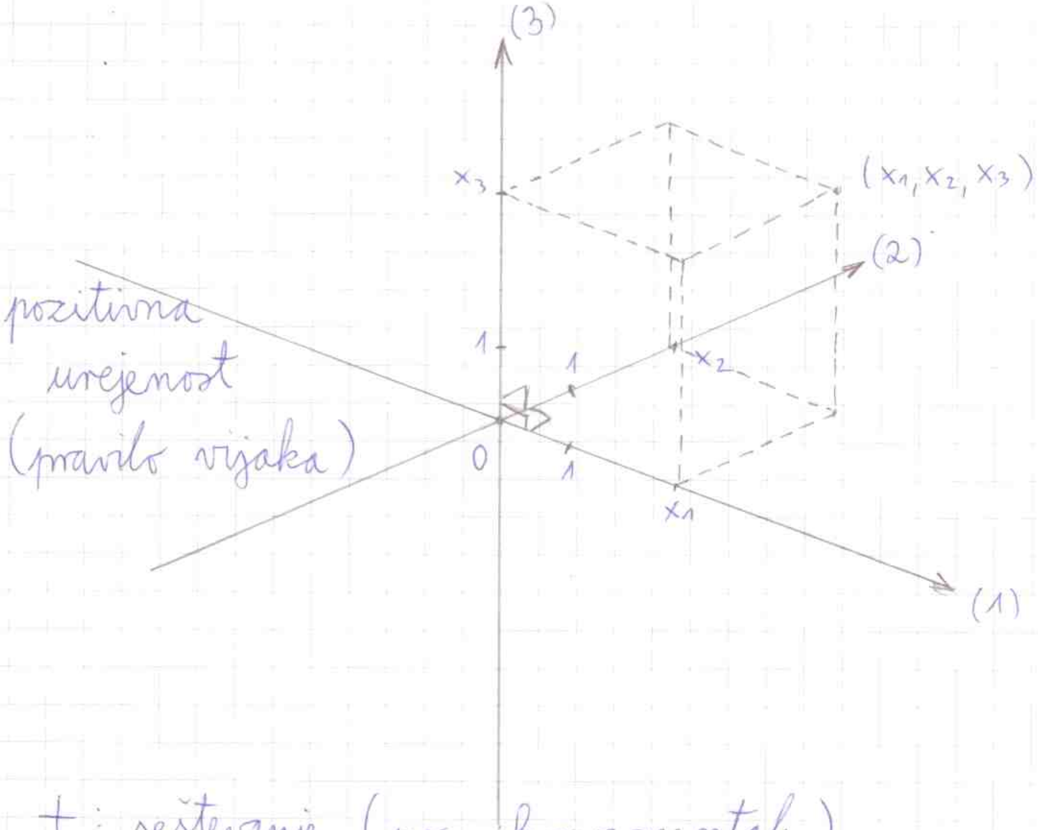
model: ravnina s koordinatnim sistemom



$\mathbb{R}^3 \equiv \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{ (x_1, x_2, x_3) : x_i \in \mathbb{R}, i = 1,2,3 \}$

1. komponenta (koordinata) ← urejena trojica

model: prostor z danim koordinatnim sistemom



+ : seštevanje (po komponentah)

$x, y \in \mathbb{R}^3$

$x = (x_1, x_2, x_3) \quad y = (y_1, y_2, y_3)$

$x + y = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$

Lastnosti: $\boxed{x + y = y + x}$ KOMUTATIVNOST

$\boxed{(x+y)+z = x+(y+z)}$ ASOCIATIVNOST

$0 = (0,0,0)$ nevtralni element za seštevanje

$\boxed{x + 0 = 0 + x = x}$

$-x = (-x_1, -x_2, -x_3)$

$\boxed{x + (-x) = 0}$ $(\mathbb{R}^3, +)$ je Abelova grupa

geometrijski aspekt:

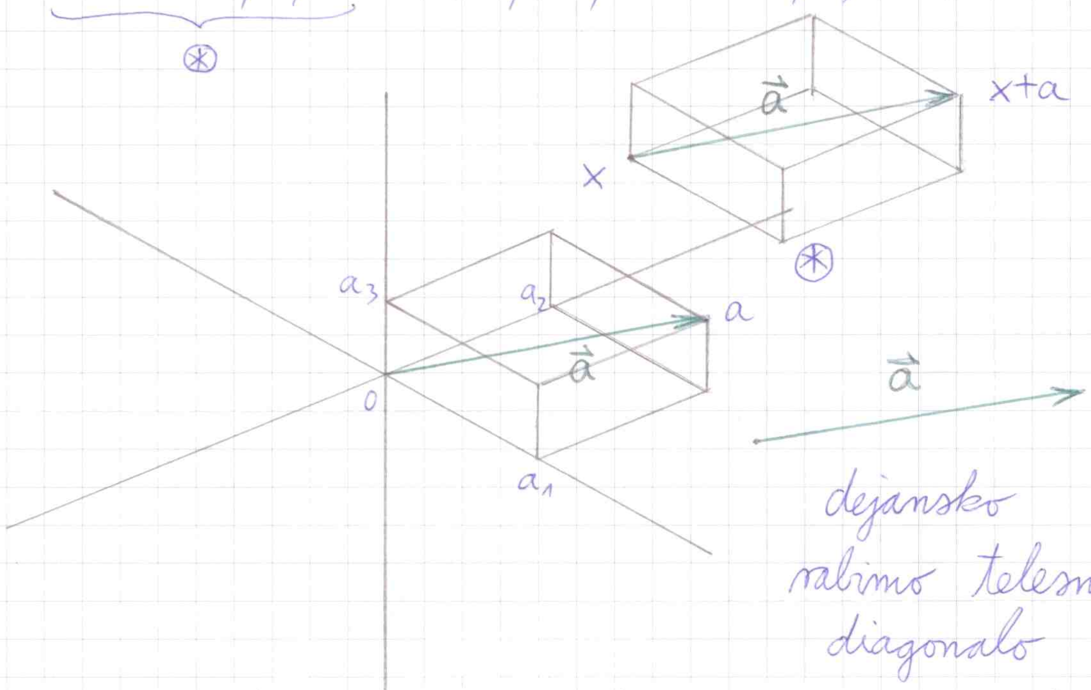
$a \in \mathbb{R}^3$ $T_a : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$T_a : x \longmapsto x + a$ $(x \in \mathbb{R}^3)$

translacija za a ali vzporedni premik

$a = (a_1, a_2, a_3)$

$$x + a = \underbrace{x + (a_1, 0, 0)}_{\circledast} + (0, a_2, 0) + (0, 0, a_3)$$



dejansko rabimo telesno diagonalo

$\vec{a}$ - vektor (je) družina vseh usmerjenih daljic od x do $x+a$ $(x \in \mathbb{R}^3)$

(usmerjene daljice $\equiv$ vektorji) krajevni vektor

$a \equiv \vec{a}$

množenje s številom (razteg)

$x \in \mathbb{R}^3$, $\alpha \in \mathbb{R}$

$x = (x_1, x_2, x_3)$

$\alpha x = (\alpha x_1, \alpha x_2, \alpha x_3)$

$0 x = 0$

skalar 0 — vektor 0

$(-1) x = -x$

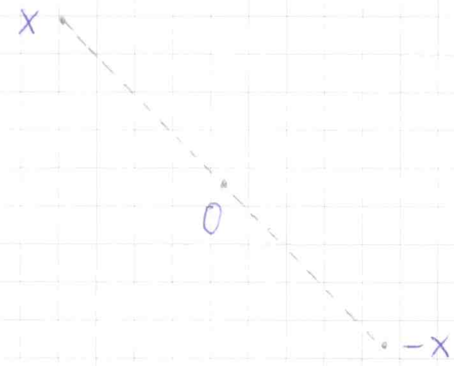
geometrijsko:

$Z : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$x \longmapsto -x$

$Zx = -x$

Z - zrcaljenje preko 0

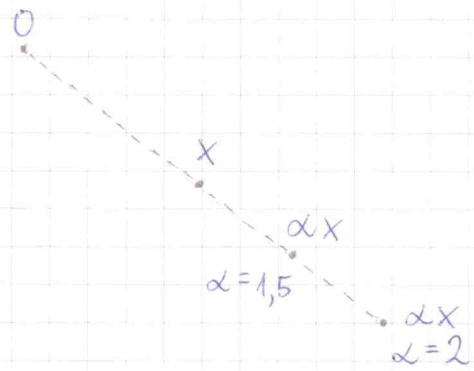


$0 < \alpha \in \mathbb{R}$

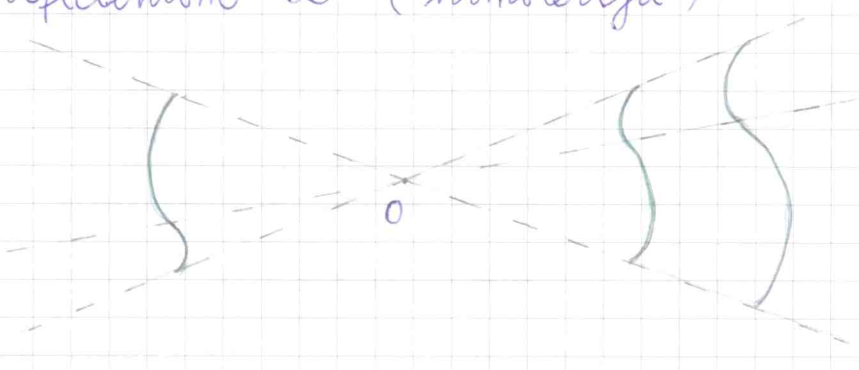
$R_\alpha : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$R_\alpha x = \alpha x$

$x \longmapsto \alpha x$



R_α - razteg s središčem 0 in koeficientom α (homotetija)



$\alpha < 0$

$R_\alpha = Z R_{-\alpha}$

razteg

Osnovna pravila:

$\alpha(x+y) = \alpha x + \alpha y \quad ; \alpha \in \mathbb{R} ; \; x, y \in \mathbb{R}^3$

$(\alpha+\beta)x = \alpha x + \beta x \quad ; \alpha, \beta \in \mathbb{R} ; \; x \in \mathbb{R}^3$

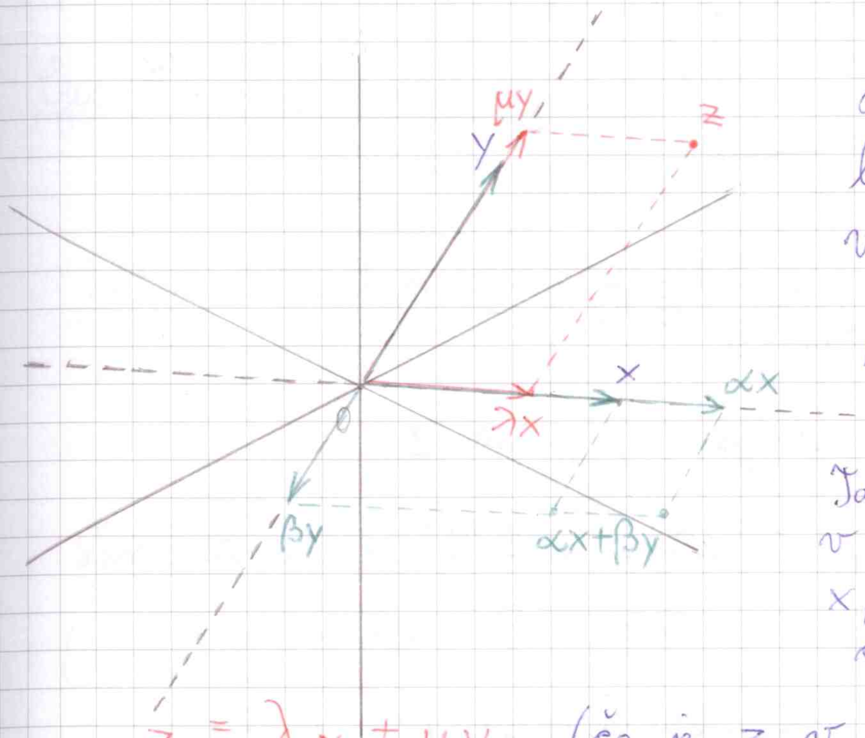
/* skalar je ponavadi levo od vektorja */

$(-1)x = -x$

$x, y \in \mathbb{R}^3$

Če je eden od obeh večkratnik drugega, sta x in y **linearno odvisna**. V nasprotnem primeru sta x in y **linearno neodvisna**.

linearno odvisna



$\alpha x + \beta y$

linearna kombinacija vektorjev x in y

$M = \{\alpha x + \beta y : \alpha, \beta \in \mathbb{R}\}$

Ta množica je vsebovana v ravnini, ki gre skozi x, y in 0. Ali pokriva celo ravnino?

$z = \lambda x + \mu y$ (če je z v ravnini, določeni z x in y)

M = ravnina skozi x, y (in 0)

to spustimo, če mislimo x in y kot vektorja

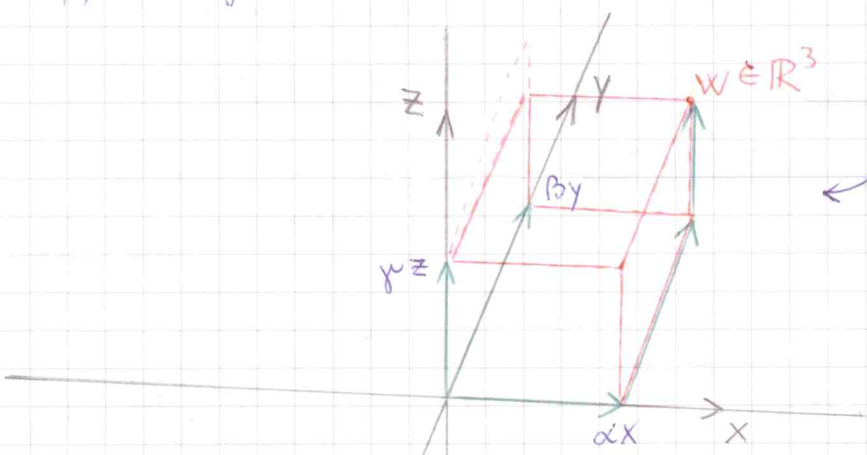
$x, y \in \mathbb{R}^3$ sta linearno neodvisna natanko takrat, kadar velja sklep:

$$\alpha x + \beta y = 0 \implies \alpha = \beta = 0$$

$x, y, z \in \mathbb{R}^3$

Če je eden od njih linearna kombinacija drugih dveh, so x, y in z linearno odvisni, sicer pa so linearno neodvisni.

x, y, z naj bodo linearno neodvisni



← paralelepiped je določen z αx, βy in γz

$$\{\alpha x + \beta y + \gamma z : \alpha, \beta, \gamma \in \mathbb{R}\} = \mathbb{R}^3$$

linearna kombinacija x, y in z

$w = \alpha x + \beta y + \gamma z$ ← vsak $w \in \mathbb{R}^3$ se da zapisati na ta način

Za vsak $w \in \mathbb{R}^3$ so ~~izrazi~~ koeficienti α, β in γ v izrazu $w = \alpha x + \beta y + \gamma z$ enolično določeni.

Velja: $x, y, z \in \mathbb{R}^3$ so linearno neodvisni natanko takrat, kadar velja sklep:

$$\alpha x + \beta y + \gamma z = 0 \implies \alpha = \beta = \gamma = 0$$

Skalarni produkt

$x, y \in \mathbb{R}^3$

$$x \cdot y = x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3$$

Osnovne lastnosti:

$$x \cdot (y + z) = x \cdot y + x \cdot z$$

$$x \cdot y = y \cdot x \qquad \alpha \in \mathbb{R}$$

$$(\alpha x) \cdot y = \alpha (x \cdot y)$$

„Uporaba":

$$(2x - y) \cdot (x + y) = (2x - y) \cdot x + (2x - y) \cdot y =$$

$$= (2x) \cdot x \underbrace{- y \cdot x + (2x) \cdot y}_{+ x \cdot y} - y \cdot y =$$

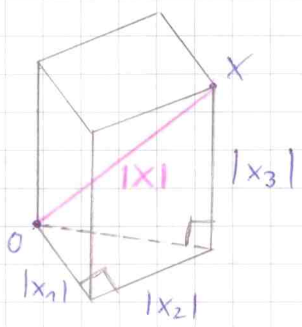
$$= 2(x \cdot x) + x \cdot y - y \cdot y$$

Geometrijska interpretacija:

$$x \cdot x = x_1^2 + x_2^2 + x_3^2 = |x|^2$$

$|x|$ – dolžina (velikost) vektorja x

$$|x| = \sqrt{x \cdot x}$$

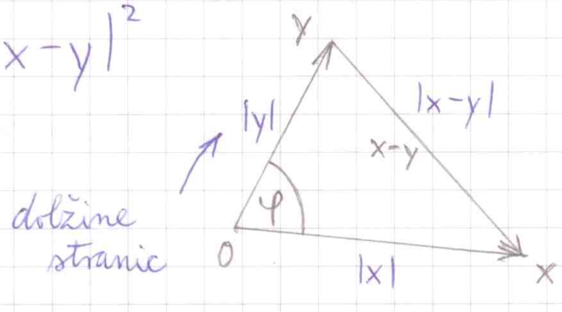


$x, y \in \mathbb{R}^3$

$$(x - y) \cdot (x - y) = x \cdot x - x \cdot y - y \cdot x + y \cdot y =$$

$$= x \cdot x - 2x \cdot y + y \cdot y = |x|^2 - 2x \cdot y + |y|^2$$

$$(x - y) \cdot (x - y) = |x - y|^2$$



Kosinusni izrek:

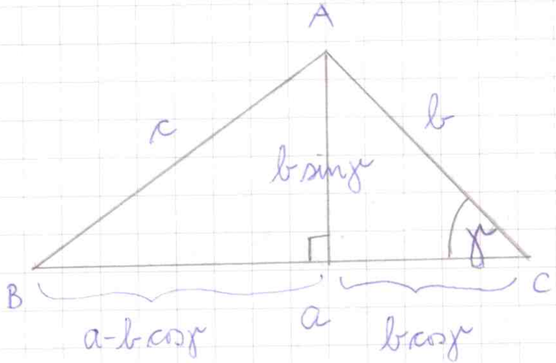
$$|x-y|^2 = |x|^2 + |y|^2 - 2|x|\cdot|y|\cdot\cos\varphi$$

$$|x-y|^2 = (x-y)\cdot(x-y) = |x|^2 - 2x\cdot y + |y|^2$$

$$x\cdot y = |x|\cdot|y|\cdot\cos\varphi$$

$\varphi = \frac{\pi}{2}$ (pravokotnost) $\quad x \perp y \iff x\cdot y = 0$

Dokaz:



$$c^2 = (a - b\cos\gamma)^2 + (b\sin\gamma)^2$$
$$c^2 = a^2 - 2ab\cos\gamma + b^2$$

$\vec{a}, \vec{b} \in \mathbb{R}^3$

$\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0$

$\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$ paroma pravokotni

$|\vec{u}| = |\vec{v}| = |\vec{w}| = 1$ tvorijo bazo v $\mathbb{R}^3$

$\{\vec{u}, \vec{v}, \vec{w}\}$ je ortonormirana baza

$\vec{i} = (1,0,0)$

$\vec{j} = (0,1,0)$

$\vec{k} = (0,0,1)$

$\{\vec{i}, \vec{j}, \vec{k}\}$ standardna baza je ortonormirana

$$x = (x_1, x_2, x_3) \in \mathbb{R}^3$$

$$x = x_1 \vec{i} + x_2 \vec{j} + x_3 \vec{k}$$

$$x, y \in \mathbb{R}^2 \qquad (\subseteq \mathbb{R}^3)$$

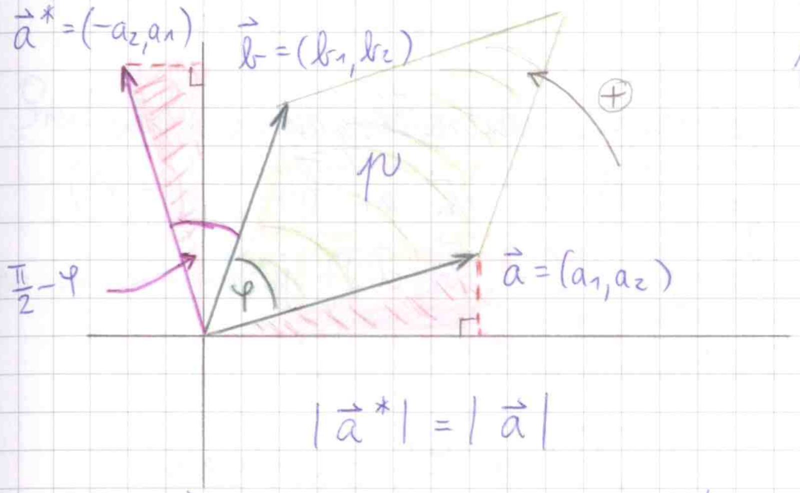
$$x = (x_1, x_2) \qquad (\equiv (x_1, x_2, 0))$$

$$y = (y_1, y_2) \qquad (\equiv (y_1, y_2, 0))$$

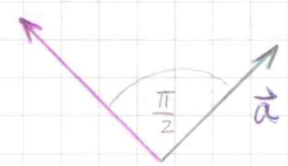
$$x \cdot y = x_1 y_1 + x_2 y_2$$

Uporaba skalarnega produkta:

ploščina paralelograma v $\mathbb{R}^2$

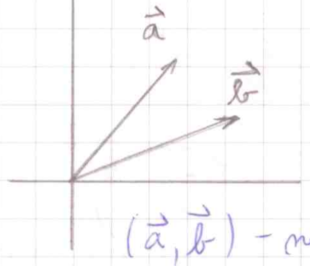


$$p = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi =$$
$$= |\vec{a}| \cdot |\vec{b}| \cdot \cos\left(\frac{\pi}{2} - \varphi\right)$$



$$|\vec{a}| \cdot |\vec{b}| \cdot \cos\left(\frac{\pi}{2} - \varphi\right) = |\vec{a}^*| |\vec{b}| \cos\left(\frac{\pi}{2} - \varphi\right) = \vec{a}^* \cdot \vec{b}$$

$(\vec{a}, \vec{b})$ - pozitivno orientiran



$(\vec{a}, \vec{b})$ - negativno orientiran

$(\vec{a}, \vec{b})$ je negativno orientiran

$\Longleftrightarrow$

$(\vec{b}, \vec{a})$ je pozitivno orientiran

$(\vec{a}, \vec{b})$ negat. orientiran $\Rightarrow$ $(\vec{b}, \vec{a})$ pozit. orientiran $\Rightarrow$

$$p = \vec{b}^* \cdot \vec{a}$$

$\vec{a}^* \cdot \vec{b} = (-a_2, a_1) \cdot (b_1, b_2) = a_1 b_2 - a_2 b_1$

$\vec{a} \cdot \vec{b}^* = (-b_2, b_1) \cdot (a_1, a_2) = -a_1 b_2 + a_2 b_1$

$$\boxed{p = \pm (a_1 b_2 - a_2 b_1)}$$

$+$ če je orient. $(\vec{a}, \vec{b})$ $\oplus$

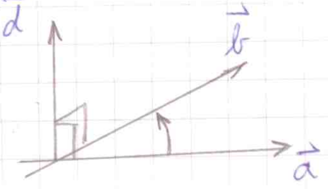
$-$ če je orient. $(\vec{a}, \vec{b})$ $\ominus$

$\begin{vmatrix} p & q \\ r & s \end{vmatrix} \equiv ps - qr \qquad p = \pm \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$

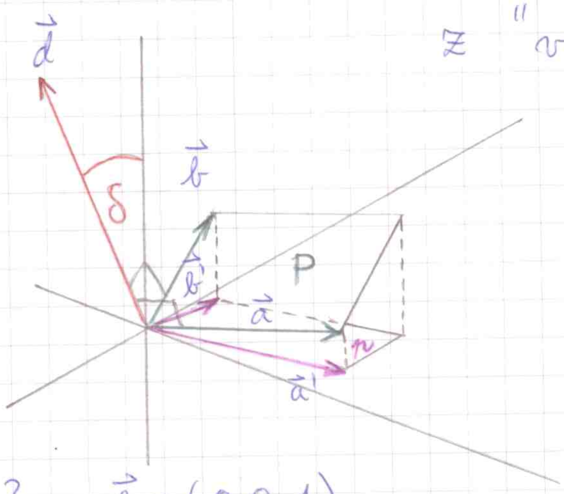
Vektorski produkt

$\vec{a}, \vec{b} \in \mathbb{R}^3 \; ; \quad \vec{d} = \vec{a} \times \vec{b} \in \mathbb{R}^3$

1. $\vec{d} \perp \vec{a}$, $\vec{d} \perp \vec{b}$
2. $|\vec{a} \times \vec{b}| = |\vec{d}|$ = ploščina paralelograma, ki ga določata $\vec{a}$ in $\vec{b}$ $(= P)$
3. Urejena trojica vektorjev $(\vec{a}, \vec{b}, \vec{d})$ je pozitivno orientirana.



$(\vec{a}, \vec{b})$ je $\oplus$ orientiran v ravnini, ki jo določata $\vec{a}$ in $\vec{b}$, gledano z "vrha" $\vec{d}$



$\left.\begin{array}{l} \vec{a}' = (a_1, a_2, 0) \\ \vec{b}' = (b_1, b_2, 0) \end{array}\right\}$

določata paralelogram s ploščino p

$d_3 = ?$ $\quad \vec{k} = (0, 0, 1)$

$[\vec{d} = (d_1, d_2, d_3)]$

δ - kot med $\vec{d}$ in $\vec{k}$

= kot med ravninama paralelograma $\vec{a}, \vec{b}$ in $\vec{a}', \vec{b}'$

$\Rightarrow \quad p = \pm P \cos\delta$

$+$, če $0 \le \delta \le \frac{\pi}{2}$

$-$, če $\frac{\pi}{2} < \delta \le \pi$

$\Rightarrow \quad p = |\vec{d}| \cos\delta$

$$d_3 = \vec{d} \cdot \vec{k} = \underset{= P}{|\vec{d}|} \cdot \underbrace{|\vec{k}|}_{=1} \cos\delta = \underset{= P}{|\vec{d}|} \cdot \cos\delta$$

$$\Rightarrow \quad d_3 = \pm p = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Predznaki se uredijo sami po sebi. Zakaj?

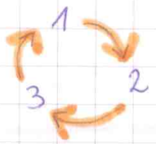
$\boxed{+ : \quad 0 \le \delta \le \frac{\pi}{2}}$ $\qquad$ $\boxed{- : \quad \frac{\pi}{2} \le \delta \le \pi}$

pri determinanti $\boxed{+ : \text{ orient. } (\vec{a}', \vec{b}') \text{ je } \oplus}$

$\boxed{- : \text{ orient. } (\vec{a}', \vec{b}') \text{ je } \ominus}$

(v ravnini $\mathbb{R}^2 \times \{0\}$)

$$\Rightarrow \quad d_3 = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$



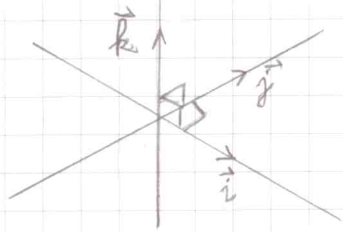
Podobno dobimo:

$$d_1 = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, \qquad d_2 = \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix} = - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}$$

$$\vec{d} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \cdot \vec{k} \equiv$$

$$\equiv \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \vec{a} \times \vec{b}$$

$\vec{i} \times \vec{j} = \vec{k}$
$\vec{i} \times \vec{k} = -\vec{j}$
$\vec{j} \times \vec{k} = \vec{i}$



Lastnosti:

antikomutativen

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

/* če ni oklepaja se nanaša na vse */

$$(\alpha \vec{a}) \times \vec{b} = \alpha (\vec{a} \times \vec{b}), \quad \alpha \in \mathbb{R}$$

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$$

$$\vec{a} \times (\beta \vec{b}) = \beta (\vec{a} \times \vec{b}) \quad \beta \in \mathbb{R}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \times \vec{a} = \vec{0}$$

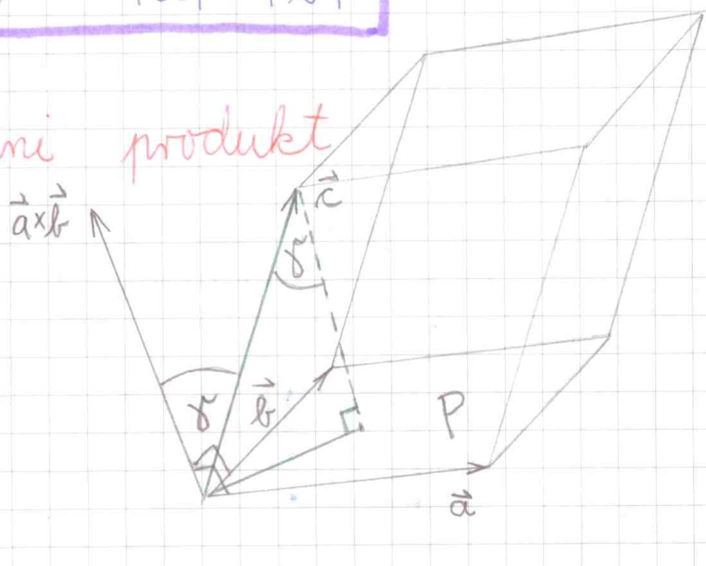
$\vec{a}, \vec{b}$ sta linearno odvisna $\iff$ $\vec{a} \times \vec{b} = \vec{0}$

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi$$
$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

$$\boxed{|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 \cdot |\vec{b}|^2}$$

Mešani produkt

$$(\vec{a} \times \vec{b}) \cdot \vec{c}$$



paralelepiped

V - njegova prostornina

v - višina

$$V = P \cdot v = \pm P \cdot |\vec{c}| \cdot \cos\gamma$$

$(\vec{a}, \vec{b}, \vec{c})$ je $\oplus$ orient.

$\Updownarrow$

$+ : \ 0 \leq \gamma \leq \frac{\pi}{2}$

$- : \ \frac{\pi}{2} \leq \gamma \leq \pi$

$\Updownarrow$

$(\vec{a}, \vec{b}, \vec{c})$ je $\ominus$ orient.

$$P = |\vec{a} \times \vec{b}| \Rightarrow$$

$$V = \pm |\vec{a} \times \vec{b}| \cdot |\vec{c}| \cdot \cos\gamma = \pm (\vec{a} \times \vec{b}) \cdot \vec{c}$$

$$\boxed{(\vec{a} \times \vec{b}) \cdot \vec{c} = \pm V}$$

$+$ ali $-$ izberemo v skladu z orientacijo $(\vec{a}, \vec{b}, \vec{c})$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{b} \times \vec{c}) \cdot \vec{a} = \vec{a} \cdot (\vec{b} \times \vec{c})$$

(sledi iz geom. interpretacije)

$$\boxed{(\vec{a} \times \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \times \vec{c})} \equiv$$

$$\equiv [\vec{a}, \vec{b}, \vec{c}]$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = a_1 (\vec{b} \times \vec{c})_1 + a_2 (\vec{b} \times \vec{c})_2 + a_3 (\vec{b} \times \vec{c})_3 \equiv$$

$$\left[\vec{b} \times \vec{c} = \begin{vmatrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad \begin{array}{l} \vec{i} \leftrightarrow a_1 \\ \vec{j} \leftrightarrow a_2 \\ \vec{k} \leftrightarrow a_3 \end{array} \right]$$

$$\equiv \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$ determinanta reda 3

Kdaj je $[\vec{a}, \vec{b}, \vec{c}] = 0$?

$V = 0 \iff \vec{a}, \vec{b}, \vec{c}$ so linearno odvisni

Vektorji $\vec{a}, \vec{b}, \vec{c}$ so linearno neodvisni $\iff$

$\iff [\vec{a}, \vec{b}, \vec{c}] \neq 0 \quad \left(\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \neq 0 \right)$

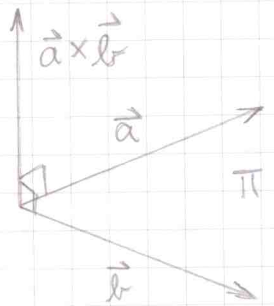
Dvojni vektorski produkt

$\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$

(*) $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{d}$ $\quad \vec{a}, \vec{b}$ linearno neodvisna

$\vec{d} \perp (\vec{a} \times \vec{b}) \quad \vec{d} \perp \vec{c}$

$\vec{d}$ leži v Π



$\Rightarrow \vec{d} = \alpha \vec{a} + \beta \vec{b}$

$\vec{d} \cdot \vec{c} = 0 \Rightarrow (\alpha \vec{a} + \beta \vec{b}) \cdot \vec{c} = 0$

$\alpha \vec{a} \cdot \vec{c} + \beta \vec{b} \cdot \vec{c} = 0$

$\beta = \lambda \vec{a} \cdot \vec{c} \qquad \alpha = -\lambda \vec{b} \cdot \vec{c}$

$\vec{d} = -\lambda (\vec{b} \cdot \vec{c}) \vec{a} + \lambda (\vec{a} \cdot \vec{c}) \vec{b}$ **(**)**

$\lambda = ?$

Primerjamo eno od komponent vektorja $\vec{d}$ v izrazih **(*)** in **(**)** $\Rightarrow$ dobimo $\lambda = 1$

$$\boxed{(\vec{a} \times \vec{b}) \times \vec{c} = -(\vec{b} \cdot \vec{c}) \vec{a} + (\vec{a} \cdot \vec{c}) \vec{b}}$$

velja tudi za linearno odvisna $\vec{a}, \vec{b}$

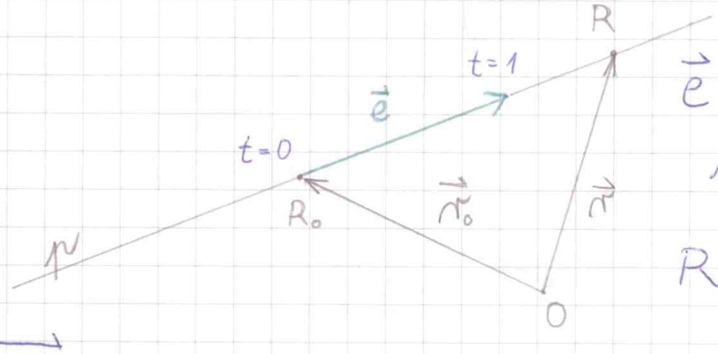
$(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$

na splošno

Kdaj pa tu je enačaj ?

„Analitična geometrija" v $\mathbb{R}^3$

– enačbe premic, ravnin



$R_0 : (x_0, y_0, z_0) = \vec{r}_0$

$\vec{e} = (a, b, c) \; (\neq \vec{0})$

smerni vektor premice p

$R : (x, y, z) = \vec{r}$

R na p

$\overrightarrow{R_0R} = t\vec{e}, \quad t \in \mathbb{R}$

$\overrightarrow{R_0R} = \vec{r} - \vec{r}_0$

$\vec{r} - \vec{r}_0 = t\vec{e}, \quad t \in \mathbb{R}$

$$\boxed{\vec{r} = \vec{r}_0 + t\vec{e}, \quad t \in \mathbb{R}}$$

po komponentah:

$$\left.\begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \\ z &= z_0 + tc \end{aligned}\right\} t \in \mathbb{R}$$

parametrična enačba

izrazimo t ...
$$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

$a = 0: \; x = x_0 \qquad b = 0: \; y = y_0$

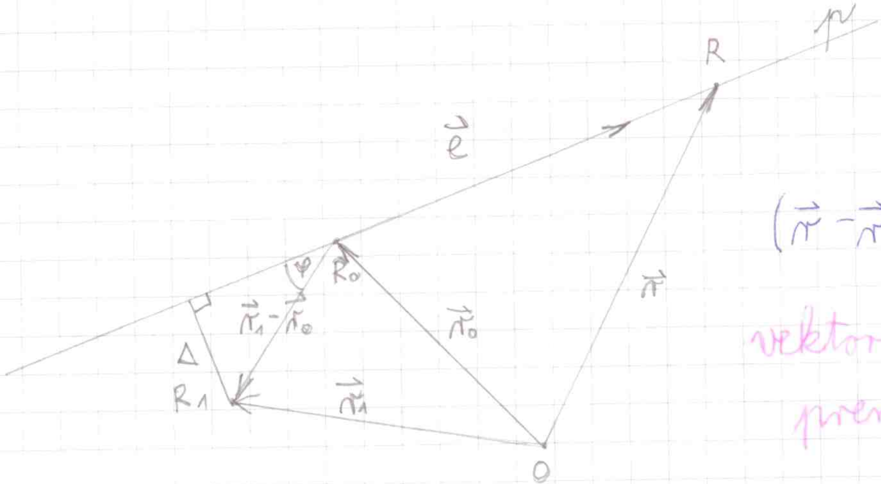
$c = 0: \; z = z_0 \qquad \left(\frac{z - z_0}{0}\right)$

Primer: $2x + 1 = y - 2 = \frac{1}{2} z$

$$\frac{x - (-\frac{1}{2})}{\frac{1}{2}} = \frac{y - 2}{1} = \frac{z - 0}{2}$$

$\vec{e} = (a, b, c) = \left(\frac{1}{2}, 1, 2\right)$

$\vec{r}_0 = (x_0, y_0, z_0) = \left(-\frac{1}{2}, 2, 0\right)$



$$(\vec{r}-\vec{r}_0)\times\vec{e}=\vec{0}$$

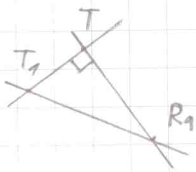
vektorska enačba premice

$$|(\vec{r}_1-\vec{r}_0)\times\vec{e}| = |\vec{r}_1-\vec{r}_0|\cdot|\vec{e}|\cdot\sin\varphi$$

$$\Delta = |\vec{r}_1-\vec{r}_0|\cdot\sin\varphi$$

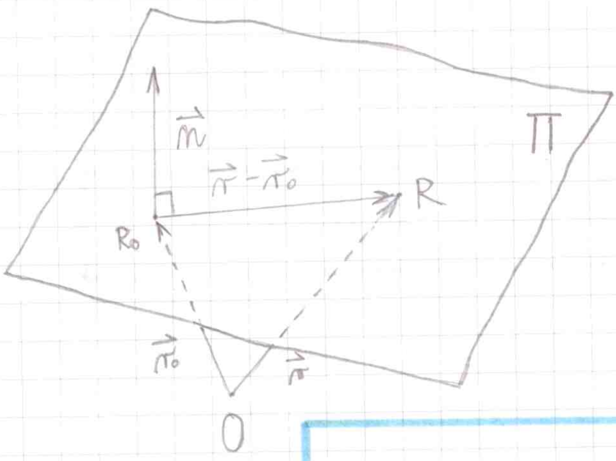
$$\boxed{\Delta = \frac{|(\vec{r}_1-\vec{r}_0)\times\vec{e}|}{|\vec{e}|}}$$

razdalja med R_1 in p



$|T_1R_1| > |TR_1|$

Enačba ravnine



$\vec{n}=(a,b,c)$

normala na Π

$(\vec{n}\neq\vec{0})$

$$\boxed{(\vec{r}-\vec{r}_0)\cdot\vec{n}=\vec{0}}$$

$\vec{r}_0=(x_0,y_0,z_0)$

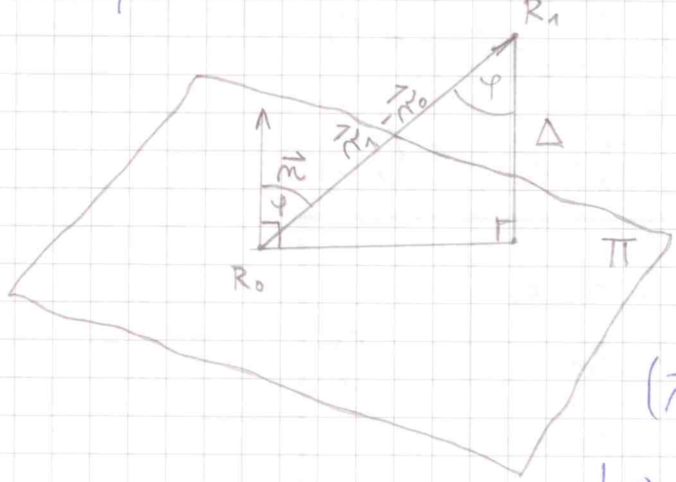
$\vec{r}=(x,y,z)$

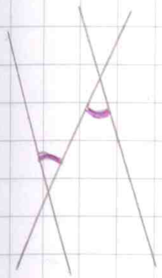
$$(x-x_0,\ y-y_0,\ z-z_0)\cdot(a,b,c)=0$$

$$\boxed{a(x-x_0)+b(y-y_0)+c(z-z_0)=0}$$

implicitna enačba ravnine $\boxed{ax+by+cz+d=0}$ $\quad d=-ax_0-by_0-cz_0$

Implicitna enačba ravnine je do konst. faktorja natančno določena, ni pa enolična.





O

$$(\vec{r}_1 - \vec{r}_0)\cdot\vec{n} =$$

$$|\vec{r}_1 - \vec{r}_0|\cdot|\vec{n}|\cdot\cos\varphi$$

$$\pm\Delta = |\vec{r}_1 - \vec{r}_0|\cdot\cos\varphi$$

$$\boxed{\pm\Delta = \frac{(\vec{r}_1 - \vec{r}_0)\cdot\vec{n}}{|\vec{n}|}}$$

predznak nam pove, na kateri strani ravnine leži točka R_1

Δ = razdalja med R_1 in Π

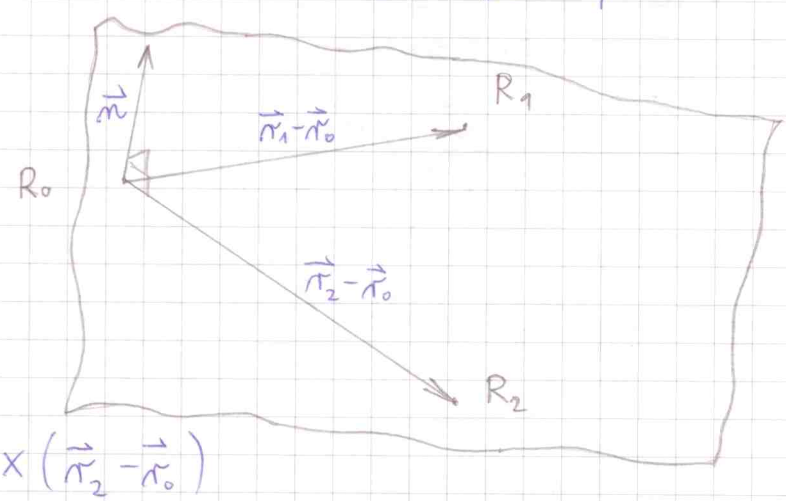
$$\Delta = \frac{|(\vec{r}_1 - \vec{r}_0)\cdot\vec{n}|}{|\vec{n}|}$$

$$|\vec{n}| = \sqrt{a^2+b^2+c^2}$$

$$\boxed{\pm\Delta = \frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}}$$

Enačba ravnine skozi tri nekolinearne točke.

niso na isti premici



$$\vec{n} = (\vec{r}_1 - \vec{r}_0)\times(\vec{r}_2 - \vec{r}_0)$$

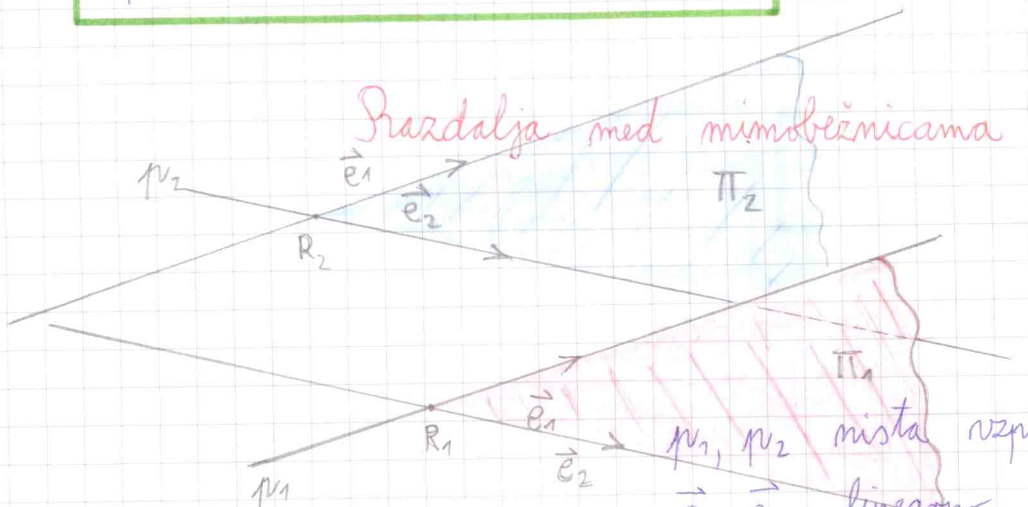
$$(\vec{r}-\vec{r}_0)\cdot\vec{n} = 0$$

$$(\vec{r}-\vec{r}_0)\cdot(\vec{r}_1-\vec{r}_0)\times(\vec{r}_2-\vec{r}_0) = 0$$

$$[\vec{r}-\vec{r}_0,\ \vec{r}_1-\vec{r}_0,\ \vec{r}_2-\vec{r}_0] = 0$$

$$\boxed{\begin{vmatrix} x-x_0, & y-y_0, & z-z_0 \\ x_1-x_0, & y_1-y_0, & z_1-z_0 \\ x_2-x_0, & y_2-y_0, & z_2-z_0 \end{vmatrix} = 0}$$

Razdalja med mimobežnicama



p_1, p_2 nista vzporedni

$\vec{e}_1, \vec{e}_2$ linearno neodvisna

$\vec{e}_1\times\vec{e}_2 \neq \vec{0}$

Δ = razdalja med p_1 in p_2

Δ = razdalja med Π_1 in Π_2 = razdalja med R_1 in Π_2

$\vec{n} = \vec{e}_1\times\vec{e}_2$ normala na Π_2 (in Π_1)

$$\Delta = \frac{|(\vec{r}_1-\vec{r}_2)\cdot(\vec{e}_1\times\vec{e}_2)|}{|\vec{e}_1\times\vec{e}_2|}$$

$$\Delta = \frac{|[\vec{r}_1-\vec{r}_2,\ \vec{e}_1,\ \vec{e}_2]|}{|\vec{e}_1\times\vec{e}_2|}$$

Kako poiščemo najbližji točki?