A

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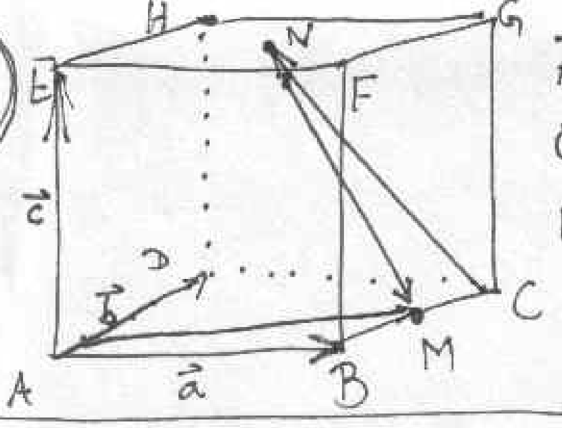

$\vec{AM} = \vec{a} + \frac{1}{2}\vec{b}$

$\vec{CN} = \vec{c} - \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$

$\vec{NM} = \frac{1}{2}\vec{a} - \vec{c}$

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2 $|\vec{a}| = \frac{\sqrt{2}}{2}$, $|\vec{b}| = 2$, $\varphi = 135^\circ$

(a) $\ldots = 9|\vec{a}|^2 + 3\vec{a}\cdot\vec{b} - 6\vec{b}\cdot\vec{a} - 2|\vec{b}|^2$

$= 9\cdot\frac{1}{2} - 3\cdot|\vec{a}|\cdot|\vec{b}|\cdot\cos 135^\circ - 2\cdot 4$

$= \frac{9}{2} - 3\cdot\frac{\sqrt{2}}{2}\cdot 2\cdot\left(-\frac{\sqrt{2}}{2}\right) - 8 = \underline{\underline{-1/2}}$

(b) $\ldots = \sqrt{(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})} = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}}$

$= \sqrt{\frac{1}{2} + 4 + 2\cdot\frac{\sqrt{2}}{2}\cdot 2\cdot\left(-\frac{\sqrt{2}}{2}\right)}$

$= \sqrt{\frac{1}{2} + 4 - 2} = \sqrt{\frac{5}{2}} = \underline{\underline{\frac{\sqrt{10}}{2}}}$

(c) $\ldots \cos\varphi = \frac{\vec{a}\cdot(\vec{a}+\vec{b})}{|\vec{a}|\cdot|\vec{a}+\vec{b}|} = \frac{|\vec{a}|^2 + \vec{a}\cdot\vec{b}}{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{10}}{2}} = \frac{\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot 2\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{20}}{4}}$

$= \frac{\frac{3}{2}}{\frac{\sqrt{20}}{4}} = \frac{6}{\sqrt{20}} = \frac{6\sqrt{20}}{20} = \underline{\underline{\frac{\sqrt{20}}{5}}}$

$\underline{\underline{\varphi = 26^\circ 34'}}$

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6. $\cos\psi = \frac{\vec{NM}\cdot\vec{NC}}{|\vec{NM}|\cdot|\vec{NC}|} = \frac{\left(\vec{a}+\frac{1}{2}\vec{b}\right)\left(\vec{c}-\frac{\vec{b}}{2}-\frac{\vec{a}}{2}\right)}{\sqrt{\frac{1}{4}+1}\cdot\sqrt{1+\frac{1}{4}+\frac{1}{4}}}$

$\left(\begin{array}{l}\vec{a}\perp\vec{b}\perp\vec{c}\\ \vec{a}\cdot\vec{b}=0\\ \vec{b}\cdot\vec{c}=0\\ \vec{a}\cdot\vec{c}=0\end{array}\right)$

$= \frac{\frac{1}{2}\vec{a}\vec{c} - \frac{1}{4}\vec{a}\cdot\vec{b} - \frac{1}{4}\vec{a}\cdot\vec{a} - \vec{c}\cdot\vec{c} + \frac{\vec{b}\vec{c}}{2} + \frac{\vec{a}\vec{c}}{2}}{\sqrt{5/4}\cdot\sqrt{6/4}}$

$= \frac{-\frac{1}{4}|\vec{a}|^2 - |\vec{c}|^2}{\sqrt{30}/4} = \frac{-5/4}{\sqrt{30}/4} = -\frac{\sqrt{30}}{6}$

$\Rightarrow \underline{\psi = 155^\circ 54'}$

$|\vec{NM}| = \sqrt{\left(\frac{1}{2}\vec{a}-\vec{c}\right)\left(\frac{1}{2}\vec{a}-\vec{c}\right)} = \sqrt{\frac{1}{4}|\vec{a}|^2 + |\vec{c}|^2 - \vec{c}\cdot\vec{a}} = \sqrt{\frac{1}{4}+1}$

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3 $A(-1,-4,5)$ $D(-2,2,-4)$

$B(5,7,3)$

$C(-4,-3,-5)$

$\vec{r}_T = \frac{1}{3}\left((-1,-4,5) + (5,7,3) + (-4,-3,-5)\right)$

$= \frac{1}{3}(0,0,3) = (0,0,1) = \vec{k}$

$T(0,0,1)$

(a) $d(T,D) = |\vec{DT}| = |\vec{r}_T - \vec{r}_D| = |(2,-2,5)| = \sqrt{4+4+25} = \underline{\underline{\sqrt{33}}}$

(b) $\vec{v}\perp\vec{r}_B \Leftrightarrow (x,5,1)\cdot(5,7,3) = 0$

$5x + 35 + 3 = 0 \Rightarrow 5x = -38$, $\underline{\underline{x = -\frac{38}{5}}}$

(c) $\vec{CM} = 4\vec{CT}$, $\vec{r}_m - \vec{r}_c = 4\cdot(\vec{r}_T - \vec{r}_c)$

$(x,y,z) - (-4,-3,-5) = 4\left((0,0,1) - (-4,-3,-5)\right)$

$x + 4 = 16 \Rightarrow x = 12$

$y + 3 = 12 \Rightarrow y = 9$

$z + 5 = 24 \Rightarrow z = 19$

$\underline{\underline{M(12,9,19)}}$

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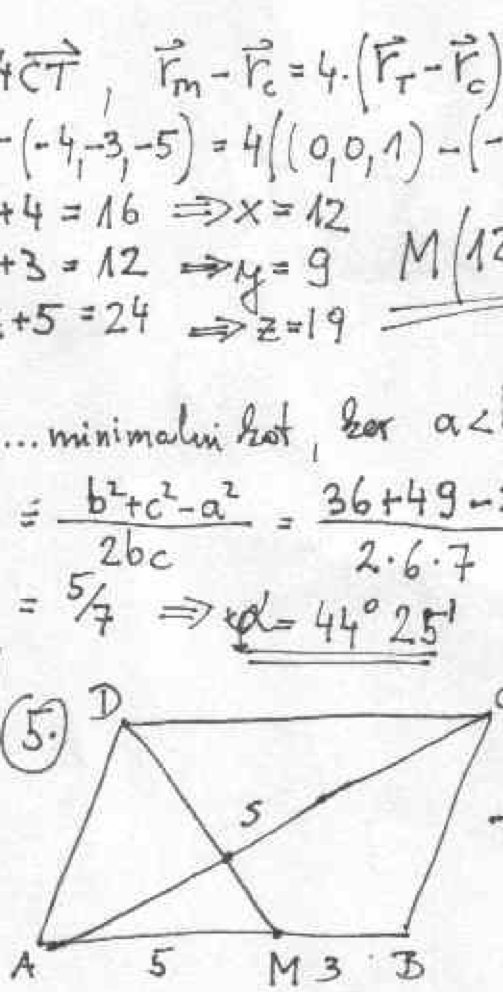
4 $\alpha\ldots$ minimalni kot, ker $a<b<c$

$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} = \frac{36+49-25}{2\cdot 6\cdot 7} = \frac{60}{12\cdot 7} = 5/7 \Rightarrow \underline{\underline{\alpha = 44^\circ 25'}}$

5.



$\vec{AB} = \vec{a}$

$\vec{AD} = \vec{b}$

$\vec{AS} = m\cdot(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

$\vec{AS} = \frac{5}{8}\vec{a} + n\left(\vec{b} - \frac{5}{8}\vec{a}\right)$

$m\vec{a} + m\vec{b} = \frac{5}{8}\vec{a} + n\vec{b} - \frac{5n}{8}\vec{a}$

$\underbrace{\left(m - \frac{5}{8} + \frac{5n}{8}\right)}_{0}\vec{a} + \underbrace{(m-n)}_{0}\vec{b} = \vec{0}$

$\begin{cases} m = n \\ 8m - 5 + 5m = 0 \end{cases}$ $13m = 5$

$m = 5/13$

$\Longrightarrow \boxed{AS : SC = 5 : 8}$