Kvadratna enačba, neenačba

1.) Nariši funkcijo tako, da bo njen graf šel skozi točko T(5,3). Vstavimo v enačbo x in y.

$$f(x)=(4-x)x^2+(5+m)x-(3m+1)$$

$$f(x)=(4-m)x^2+(5+m)x-(3m+1)$$
$$3=(4-m)\cdot 5^2+(5+m)\cdot 5-(3m+1)$$
$$3=100-25m+25+5m-3m-1$$
$$3=124-23m$$
$$23m=121$$

$$\underline{\underline{m=\frac{121}{23}}}$$

$$f(x)=(4-\frac{121}{23})x^2+(5+\frac{121}{23})x-(3\cdot\frac{121}{23}+1)$$
$$f(x)=(\frac{92}{23}-\frac{121}{23})x^2+(\frac{115}{23}+\frac{121}{23})x-(\frac{363}{23}+\frac{23}{23})$$

$$\underline{\underline{f(x)=-\frac{29}{23}x^2+\frac{236}{23}x-\frac{386}{23}}}$$

$$D=b^2-4ac$$
$$D=\frac{55696}{529}-\frac{44776}{529}$$
$$\underline{\underline{D=\frac{10920}{529}}}$$

$$1)T(p,q)$$
$$2)x_1=?\ \ x_2=?$$

$$p=\frac{-b}{2a}=\frac{-\frac{236}{23}}{2\cdot(-\frac{29}{23})}=\frac{-\frac{236}{23}}{-\frac{58}{23}}=\underline{\underline{\frac{236}{58}}}$$

$$q=\frac{-D}{4a}=\frac{-\frac{10920}{529}}{-\frac{116}{23}}=-\frac{10920}{529}\cdot\left(-\frac{116}{23}\right)=\underline{\underline{\frac{10920}{2268}}}$$

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$$

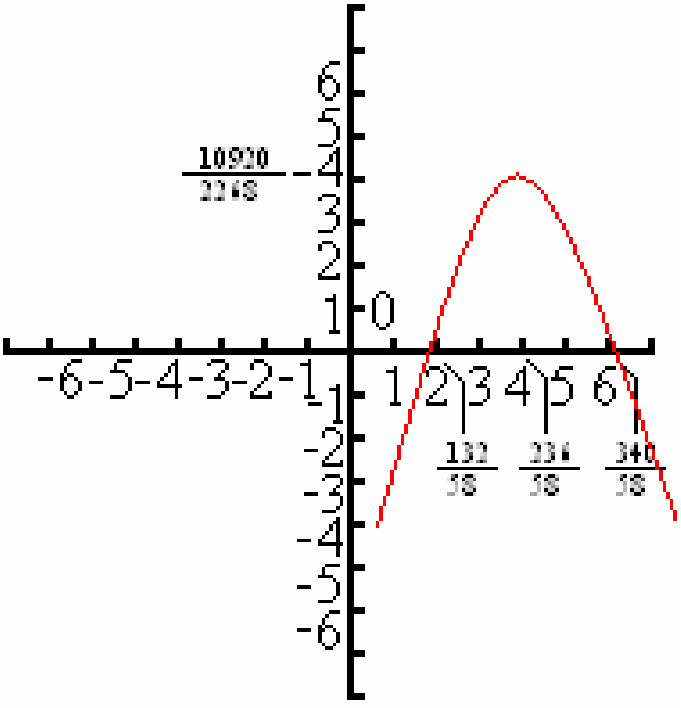
$$x_1=\frac{-\frac{236}{23}-\sqrt{\frac{10920}{529}}}{-\frac{58}{23}}=\frac{-\frac{236}{23}-\frac{104}{23}}{-\frac{58}{23}}=\underline{\underline{\frac{340}{58}}}$$

$$x_2=\frac{-\frac{236}{23}+\frac{104}{23}}{-\frac{58}{23}}=\underline{\underline{\frac{132}{58}}}$$

$a<0\Rightarrow\cap$

$\underline{\underline{1)T\left(\frac{236}{58},\frac{10920}{2268}\right)}}$

$\underline{\underline{2)\,x_1=\frac{340}{58}}}\quad\underline{\underline{x_2=\frac{10920}{2268}}}$



2.) Iz množice funkcij določi tisto, ki ima ničlo pri $x_1=5$ in nariši njen graf. Pogoj D=0. Vstavimo v enačbo x_1.

$$y=(m-1)x^2-(m+4)x+m+3$$

$$y=(m-1)x^2-(m+4)x+m+3$$
$$0=(m-1)\cdot 5^2-(m+4)\cdot 5+m+3$$
$$0=25m-25-5m-20+m+3$$
$$0=21m-42$$
$$-21m=-42$$

$$\underline{\underline{m=2}}$$

$$y=(2-1)x^2-(2+4)x+2+3$$
$$y=2x^2-x^2-2x-4x+2+3$$
$$y=x^2-6x+5$$

$$1)T(p,q)$$
$$2)x_1=?\quad x_2=?$$

$$D=b^2-4ac$$
$$D=36-4\cdot 1\cdot 5$$
$$\underline{\underline{D=16}}$$

$$p=\frac{-b}{2a}=\frac{6}{2}=\underline{\underline{3}}$$

$$q=\frac{-D}{4a}=\frac{-16}{4}=\underline{\underline{-4}}$$

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$$

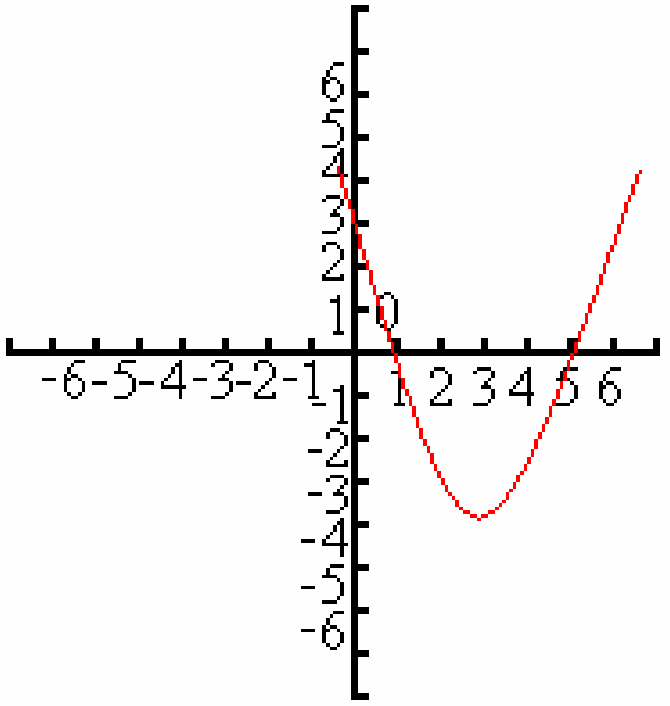
$$x_1=\frac{6-4}{2}=\frac{2}{2}=\underline{\underline{1}}$$

$$x_2=\frac{6+4}{2}=\frac{10}{2}=\underline{\underline{5}}$$

$a > 0 \Rightarrow \cup$

$1)T(3,-4)$

$2)x_1 = 1 \quad x_2 = 5$



3.) Določi *m*, da abscisna os in parabola nimata skupnih točk. Pogoj: D<0

$$y = 3mx^2 + 4x + 2$$

$$D = b^2 - 4 \cdot a \cdot c$$

$$0 > 4^2 - 4 \cdot 3 \cdot m \cdot 2$$

$$0 > 16 - 24m$$

$$m > \frac{16}{24} = \frac{2}{3}$$

$$m > \frac{2}{3}$$

4.) V polkrog s premerom 14 cm včrtaj trapez z največjim obsegom, tako, da bo premer kroga kar trapezova večja osnovnica. Kolikšne so stranice trapeza?

$$o = a + b + c$$

$$f(x) = 14 + 2b + \frac{a^2 + 4r^2 - 4b^2}{2a}$$

$$f(x) = 14 + 2b + \frac{392}{28} - \frac{4b^2}{28}$$

$$f(x) = -\frac{1}{7}b^2 + 2b + 28$$

$$v^2 = r^2 - \frac{c^2}{4}$$

$$b^2 = \frac{a^2 + c^2 - 2ac - c^2}{4} + r^2$$

$$4b^2 = a^2 - 2ac + 4r^2$$

$$c = \frac{a^2 + 4r^2 - 4b^2}{2a}$$

$$r = \frac{-b}{2a} = \frac{2}{\frac{2}{7}} = \underline{\underline{7}}$$

$$q = \frac{-D}{4a} = \frac{-(b^2 - 4ac)}{4a} = \frac{4 + 16}{\frac{4}{7}} = \underline{\underline{35}}$$

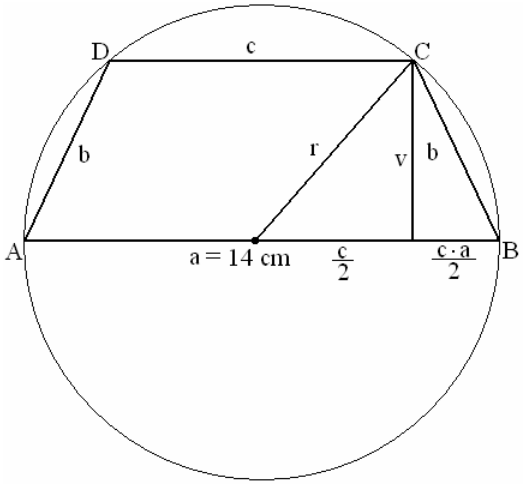
$$o = a + 2b + c$$

$$c = o - a - 2b$$

$$c = 35 - 14 - 14$$

$$\underline{\underline{c = 7}}$$

$$\underline{\underline{b = 7}} \quad \underline{\underline{c = 7}}$$



5.) Dokaži, da za poljubni dve vrednosti x_1 in x_2 funkcije $f(x)=x^2$ velja.

$$f\left(\frac{x_1+x_2}{2}\right) < \frac{1}{2}\left(f(x_1)+f(x_2)\right)$$

$$f\left(\frac{x_1+x_2}{2}\right)^2 = \frac{x_1^{\,2}+2x_1x_2+x_2^{\,2}}{4}$$

$$\frac{1}{2}\left(f(x_1)+f(x_2)\right) = \frac{1}{2}\left(x_1^{\,2}+x_2^{\,2}\right)$$

$$\frac{x_1^{\,2}+2x_1x_2+x_2^{\,2}}{4} < \frac{x_1^{\,2}+x_2^{\,2}}{2} \;/\cdot 4$$

$$(x_1+x_2)^2 < 2(x_1+x_2)^2 - 4x_1x_2$$

$$0 < x_1^{\,2} - 2x_1x_2 + x_2^{\,2}$$

$$\underline{\underline{0 < (x_1^{\,2}-x_2^{\,2})^2}}$$

6.) Reši enačbo.

(a)

$$(2x-1)^2+(x-3)^2-(x-1)\cdot(x+2)-5=0$$

$$4x^2-4x+1+x^2-6x+9-x^2-x+2-5=0$$

$$4x^2-11x+7=0$$

$$x_{1,2} = \frac{-b\pm\sqrt{D}}{2a} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x_1 = \frac{11-\sqrt{121-4\cdot 4\cdot 7}}{8} = \frac{11-\sqrt{9}}{8} = \frac{11-3}{8} = \underline{\underline{1}}$$

$$x_2 = \frac{11+\sqrt{9}}{8} = \frac{11+3}{8} = \frac{14}{8} = \underline{\underline{\frac{7}{4}}}$$

(b)

$$(x-1)^3+(2x-1)\cdot(x-3)-(x+2)^3+3x+26=0$$

$$x^3-3x^2+3x-1+2x^2-6x-x+3-x^3-6x^2-12x-8+3x+26=0$$

$$-7x^2-13x+20=0$$

$$D = b^2-4ac = 169-4\cdot(-7)\cdot(20) = \underline{\underline{729}}$$

$$x_1 = \frac{-b+\sqrt{D}}{2a} = \frac{13+27}{-14} = \frac{40}{-14} = \underline{\underline{-\frac{20}{7}}}$$

$$x_2 = \frac{-b-\sqrt{D}}{2a} = \frac{13-27}{-14} = \frac{-14}{-14} = \underline{\underline{1}}$$

(d)

$$5(x+\frac{1}{x})^2-16(x+\frac{1}{x})-52=0$$

$$t=(x+\frac{1}{x})$$

$$5t^2-16t-52=0$$

$$D=b^2-4ac=256-4\cdot(5)\cdot(-52)=\sqrt{1296}=\underline{\underline{36}}$$

$$t_1=\frac{-b+\sqrt{D}}{2a}=\frac{16+36}{10}=\frac{52}{10}=\underline{\underline{\frac{26}{5}}}$$

$$t_2=\frac{-b-\sqrt{D}}{2a}=\frac{16-36}{10}=\frac{-20}{10}=\underline{\underline{-2}}$$

$$t_1=\frac{26}{5}\Rightarrow\frac{26}{5}=x+\frac{1}{x}=/\cdot 5$$

$$26=5x+\frac{5}{x}=$$

$$26=\frac{5x^2}{x}+\frac{5}{x}=$$

$$0=5x^2-26x+5$$

$$D=b^2-4ac=676-4\cdot 5\cdot 5=\sqrt{576}=\underline{\underline{24}}$$

$$x_1=\frac{-b+\sqrt{D}}{2a}=\frac{26+24}{10}=\frac{50}{10}=\underline{\underline{5}}$$

$$x_2=\frac{-b-\sqrt{D}}{2a}=\frac{26-24}{10}=\underline{\underline{\frac{1}{5}}}$$

$$t_2=-2\Rightarrow -2=x+\frac{1}{x}=$$

$$-2=\frac{x^2}{x}+\frac{1}{x}=/\cdot x$$

$$-2x=x^2+1$$

$$0=x^2+2x+1$$

$$0=(x+1)\cdot(x+1)$$

$$\underline{\underline{x_1=-1}}\quad\underline{\underline{x_2=-1}}$$

$$\underline{\underline{x\in\{5,\frac{1}{5},-1\}}}$$

(e)

$$(\frac{2x^2-1}{x})^2-4(\frac{2x^2-1}{x})+3=0$$

$$t=(\frac{2x^2-1}{x})$$

$$t^2-4t+3=0$$

$$(t-1)\cdot(t-3)=0$$

$$t_1=1 \quad x_2=3$$

$$t_1=1\Rightarrow 1=\frac{2x^2-1}{x}=/\cdot x$$

$$x=2x^2-1$$

$$0=2x^2-x-1$$

$$D=b^2-4ac=1-4\cdot 2\cdot(-1)=\sqrt{9}=\underline{\underline{3}}$$

$$x_1=\frac{-b+\sqrt{D}}{2a}=\frac{1+3}{4}=\frac{4}{4}=\underline{\underline{1}}$$

$$x_2=\frac{1-3}{4}=\frac{-2}{4}=\underline{\underline{-\frac{1}{2}}}$$

$$t_2=3\Rightarrow 3=\frac{2x^2-1}{x}=/\cdot x$$

$$3x=2x^2-1$$

$$0=2x^2-3x-1$$

$$D=b^2-4ac=9-4\cdot 2\cdot(-1)=\underline{\underline{\sqrt{17}}}$$

$$x_1=\frac{-b+\sqrt{D}}{2a}=\underline{\underline{\frac{3+\sqrt{17}}{4}}}$$

$$x_2=\underline{\underline{\frac{3-\sqrt{17}}{4}}}$$

7.) Določi m, da bosta korena enačbe enaka. Pogoj D=0.

(a)

$$4x^2 - 8mx + 4m - 1 = 0,\ D = 0$$

$$D = b^2 - 4ac$$
$$0 = b^2 - 4ac$$
$$0 = 64m^2 - 64m + 16\ /:16$$
$$0 = 4m^2 - 4m + 1$$

$$D = b^2 - 4ac = 16 - 4 \cdot 4 \cdot 1 = \underline{\underline{0}}$$

$$m_1 = \frac{-b + \sqrt{D}}{2a} = \frac{4 + \sqrt{0}}{8} = \underline{\underline{\frac{1}{2}}}$$
$$m_2 = \frac{-b - \sqrt{D}}{2a} = \frac{4 - \sqrt{0}}{8} = \underline{\underline{\frac{1}{2}}}$$

(b)

$$x^2 - (m+3)x + 3m = 0,\ D = 0$$

$$D = b^2 - 4ac$$
$$0 = m^2 + 6m + 9 - 4 \cdot 1 \cdot 3m$$
$$0 = m^2 - 6m + 9$$
$$(m-3) \cdot (m-3)$$

$$\underline{\underline{m_1 = 3}}\ \ \underline{\underline{m_2 = 3}}$$

(c)

$$(a-2)x^2 - 2ax + a + 3 = 0,\ D = 0$$

$$D = b^2 - 4ac$$
$$0 = 4a^2 - 4 \cdot (a-2) \cdot (a+3)$$
$$0 = 4a^2 - 4a^2 - 4a + 24$$
$$0 = -4a + 24$$
$$4a = 24$$

$$\underline{\underline{a = 6}}$$

8.) Določi *m* / *a*, da bosta korena enačbe realna. Pogoj D $\geq$ 0

(a)

$$(a+1)x^2-(6a+1)x+9a+4=0,\ D\geq 0$$

$$D=b^2-4ac$$
$$36a^2+12a+1-4\cdot(9a^2+13a+4)\geq 0$$
$$36a^2+12a+1-36a^2-52a-16\geq 0$$
$$-40a-15\geq 0\ /\cdot(-1)$$
$$40a+15\leq 0$$
$$40a\leq -15$$

$$\underline{\underline{a\leq -\frac{3}{8}}}$$

(b)

$$(m+1)x^2-(2m-3)x+m-1=0,\ D\geq 0$$

$$D=b^2-4ac$$
$$4m^2-12m+9-4\cdot(m+1)\cdot(m-1)\geq 0$$
$$4m^2-12m+9-4m^2+4\geq 0$$
$$-16m+13\geq 0\ /\cdot(-1)$$
$$16m-13\leq 0$$
$$16m\leq 13$$

$$\underline{\underline{m\leq \frac{13}{16}}}$$

9.) Določi *m*, da bo razlika korenov enačbe enaka 11.

$$x^2 + (m+5)x + 3m - 4 = 0$$

$$x_1 = \frac{-b+\sqrt{D}}{2a} = \frac{-m-5+\sqrt{b^2-4ac}}{2} =$$
$$= \underline{\underline{\frac{-m-5+\sqrt{m^2-2m+41}}{2}}}$$

$$x_2 = \frac{-b+\sqrt{D}}{2a} = \frac{-m-5-\sqrt{b^2-4ac}}{2} =$$
$$= \underline{\underline{\frac{-m-5-\sqrt{m^2-2m+41}}{2}}}$$

$$\sqrt{m^2-2m+41} = 11$$

$$m^2 - 2m + 41 = 11^2$$
$$m^2 - 2m + 41 = 121$$
$$m^2 - 2m - 80 = 0$$
$$(m-10)\cdot(m+8) = 0$$

$$\underline{\underline{m_1 = 10}} \quad \underline{\underline{m_2 = -8}}$$

10.) V enačbi določi *m*, da no en koren enačbe za 3 večji od drugega.

$$x^2 - (m+1)x + m + 18 = 0$$

$$x_1 = x_2 + 3$$
$$x_1 \cdot x_2 = \frac{c}{a}$$
$$x_1 + x_2 = -\frac{b}{a}$$
$$x_2 + 3 + x_2 = -\frac{b}{a}$$
$$x_2 = -\frac{b}{2a} - \frac{3}{2}$$

$$x_2^2 + 3x_2 = \frac{c}{a}$$

$$\frac{b^2}{4a^2} + \frac{3b}{2a} + \frac{9}{4} - \frac{3b}{2a} - \frac{9}{2} = \frac{c}{a}$$

$$\frac{m^2 + 2m + 1}{4} + \frac{9}{4} - \frac{9}{2} = \frac{m + 18}{1} \quad / \cdot 4$$

$$m^2 + 2m + 1 - 9 = 4m + 72$$

$$m^2 - 2m - 80 = 0$$

$$(m - 10) \cdot (m + 8) = 0$$

$$\underline{\underline{m_1 = 10}} \quad \underline{\underline{m_2 = -8}}$$

11.) V enačbi določi *m*, da bo en koren enačbe dvakrat večji kot drugi.

$$x^2 - mx + m^2 - m = 0$$

$$x_1 = 2x_2$$

$$x_1 + x_2 = -\frac{b}{a}$$

$$3x_2 = -\frac{b}{a}$$

$$x_2 = -\frac{b}{3a}$$

$$x_1 \cdot x_2 = \frac{c}{a}$$

$$2x_2^2 = \frac{c}{a}$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}$$

$$\frac{2m^2}{9} = \frac{m^2 - m}{1} \quad / \cdot 9$$

$$2m^2 = 9m^2 - 9m$$

$$-7m^2 + 9m = 0$$

$$-m(7m - 9) = 0$$

$$\underline{\underline{m_1 = 0}} \quad \underline{\underline{m_2 = \frac{9}{3}}}$$

12.) Določi v enačbi parameter m, tako, da bo med korenoma enačbe veljala zveza $x_1^2+x_2^2=58$.

$$x^2+(m-1)x-(4m+1)=0$$

$$x_1^2+x_2^2=58$$

$$x_1+x_2=-\frac{b}{a}=\frac{-m+1}{1}=\underline{\underline{-m+1}}$$

$$x_2\cdot x_2=\frac{c}{a}=\frac{-4m-1}{1}=\underline{\underline{-4m-1}}$$

$$(x_1+x_2)^2-2x_1x_2=58$$

$$(-m+1)^2-2\cdot(-4m-1)=58$$

$$m^2-2m+1+8m+2=58$$

$$m^2+6m+3=58$$

$$m^2+6m-55=0$$

$$(m+11)\cdot(m-5)=0$$

$$\underline{\underline{m_1=-11}}\quad\underline{\underline{m_2=5}}$$

13.) določi skupne točke krivuljam in nariši grafe funkcij.

(a)

$$y=x^2-x-12$$

$$y=-3x^2+15x-12$$

$$x^2-x-12=-3x^2+15x-12$$

$$4x^2-16x=0\,/:4$$

$$x^2-4x=0$$

$$x(x-4)=0$$

$$\underline{\underline{x_1=4}}\quad\underline{\underline{x_2=0}}$$

$$x_1=4\Rightarrow y=4^2-4-12$$

$$y=16-4-12$$

$$\underline{\underline{y=0}}$$

$$x_2=0\Rightarrow y=0^2-0-12$$

$$\underline{\underline{y=-12}}$$

$$R:\underline{\underline{P_1(4,0)}}\quad\underline{\underline{P_2(0,-12)}}$$

$y = x^2 - x - 12$

$D = b^2 - 4ac$

$D = 1 - 4 \cdot (1) \cdot (-12)$

$D = 49$

$1)T(p,q)$

$2)x_1 = ? \quad x_2 = ?$

$p = \frac{-b}{2a}$

$p = \frac{1}{2} = \underline{\underline{\frac{1}{2}}}$

$q = \frac{-D}{4a}$

$q = \frac{-49}{4} = \underline{\underline{-\frac{49}{4}}}$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$x_1 = \frac{1 + \sqrt{49}}{2} = \frac{1+7}{2} = \underline{\underline{4}}$

$x_2 = \frac{1 - \sqrt{49}}{2} = \frac{1-7}{2} = \underline{\underline{-3}}$

$a > 0 \Rightarrow \cup$

$\underline{\underline{1)T(\frac{1}{2}, -\frac{49}{4})}}$

$\underline{\underline{2)x_1 = 4}} \quad \underline{\underline{x_2 = -3}}$

$y = -3x^2 + 15x - 12$

$D = b^2 - 4ac$

$D = 225 - 4 \cdot (-3) \cdot (-12)$

$D = 81$

$1)T(p,q)$

$2)x_1 = ? \quad x_2 = ?$

$p = \frac{-b}{2a}$

$p = \frac{-15}{-6} = \underline{\underline{\frac{15}{6}}}$

$q = \frac{-D}{4a}$

$q = \frac{-81}{-12} = \frac{81}{12} = \underline{\underline{\frac{27}{4}}}$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

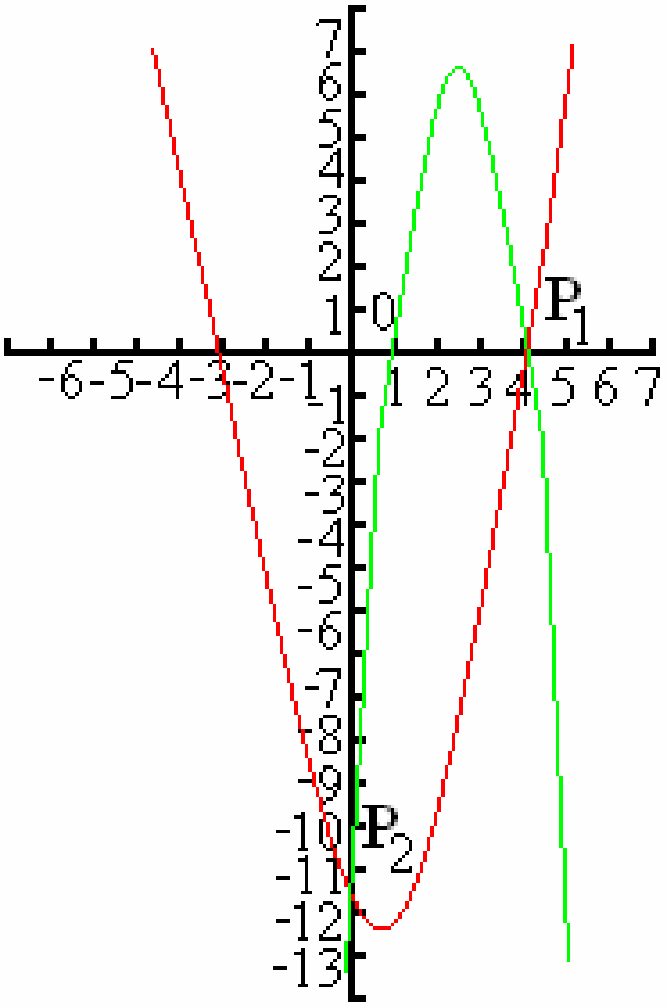
$x_1 = \frac{-15 + \sqrt{81}}{-6} = \frac{-15+9}{-6} = \frac{-6}{-6} = \underline{\underline{1}}$

$x_2 = \frac{-15 - \sqrt{81}}{-6} = \frac{-15-9}{-6} = \underline{\underline{4}}$

$a < 0 \Rightarrow \cap$

$\underline{\underline{1)T(\frac{15}{6}, \frac{27}{4})}}$

$\underline{\underline{2)x_1 = 1}} \quad \underline{\underline{x_2 = 4}}$



(b)

$$y = 2x^2 + 2x - 4$$
$$y = -x^2 - 4x + 5$$

$$2x^2 + 2x - 4 = -x^2 - 4x + 5$$
$$3x^2 + 6x - 9 = 0 \,/\, :3$$
$$x^2 + 2x - 3 = 0$$
$$(x+3)\cdot(x-1) = 0$$

$$\underline{\underline{x_1 = -3}} \quad \underline{\underline{x_2 = 1}}$$

$$x_1 = -3 \Rightarrow y = 2\cdot(-3)^2 + 2\cdot(-3) - 4$$
$$y = 18 - 6 - 4$$
$$\underline{\underline{y = 8}}$$

$$x_2 = 1 \Rightarrow y = 2 \cdot 1^2 + 2 \cdot 1 - 4 = \underline{\underline{0}}$$

$$R : \underline{\underline{P_1(-3,8)}} \quad \underline{\underline{P_2(1,0)}}$$

$y = 2x^2 + 2x - 4$

$D = b^2 - 4ac$
$D = 4 - 4 \cdot (2) \cdot (-4)$
$D = 36$

$1)T(p,q)$
$2)x_1 = ? \quad x_2 = ?$

$$p = \frac{-b}{2a}$$
$$p = \frac{-2}{4} = \underline{\underline{-\frac{1}{2}}}$$

$$q = \frac{-D}{4a}$$
$$q = \frac{-36}{8} = \underline{\underline{-\frac{9}{2}}}$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$
$$x_1 = \frac{-2 + \sqrt{36}}{4} = \frac{-2+6}{4} = \frac{4}{4} = \underline{\underline{1}}$$
$$x_2 = \frac{-2 - \sqrt{36}}{4} = \frac{-2-6}{4} = \frac{-8}{4} = \underline{\underline{-2}}$$

$a > 0 \Rightarrow \cup$
$\underline{\underline{1)T(-\frac{1}{2},-\frac{9}{2})}}$
$\underline{\underline{2)x_1 = 1}} \quad \underline{\underline{x_2 = -2}}$

$y = -x^2 - 4x + 5$

$D = b^2 - 4ac$
$D = 16 - 4 \cdot (-1) \cdot (5)$
$D = 36$

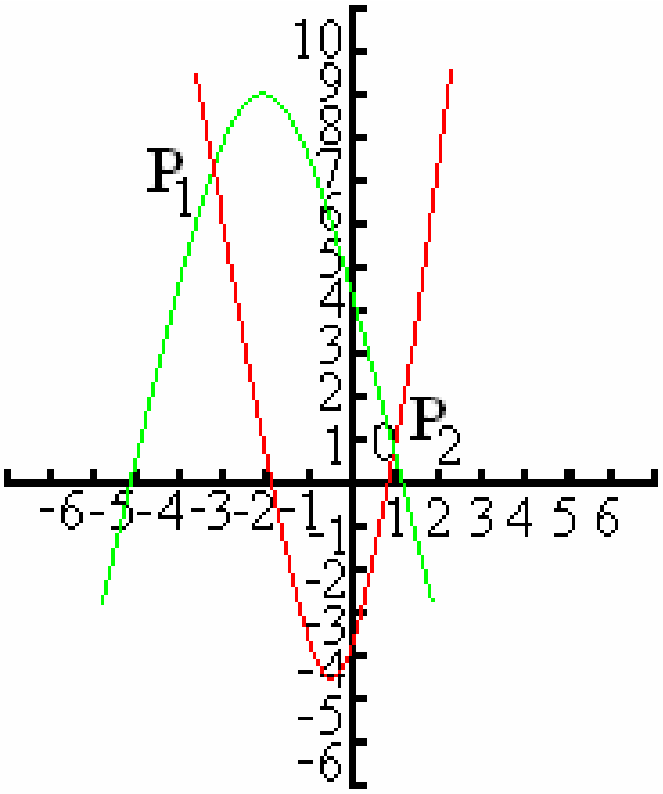
$1)T(p,q)$
$2)x_1 = ? \quad x_2 = ?$

$$p = \frac{-b}{2a}$$
$$p = \frac{4}{-2} = \underline{\underline{-2}}$$

$$q = \frac{-D}{4a}$$
$$q = \frac{-36}{-4} = \underline{\underline{9}}$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$
$$x_1 = \frac{4 + \sqrt{36}}{-2} = \frac{4+6}{-2} = \underline{\underline{-5}}$$
$$x_2 = \frac{4 - \sqrt{36}}{-2} = \frac{4-6}{-2} = \underline{\underline{1}}$$

$a < 0 \Rightarrow \cap$
$\underline{\underline{1)T(-2,9)}}$
$\underline{\underline{2)x_1 = -5}} \quad \underline{\underline{x_2 = 1}}$


P1
P2
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

(c)

$$y = -x^2 - 3x + 4$$
$$y = -x + 2$$

$$-x^2 - 3x + 4 = -x + 2$$
$$-x^2 - 2x + 2 = 0$$

$$D = b^2 - 4ac$$
$$D = 4 - 4 \cdot (-1) \cdot (2) = \underline{\underline{12}}$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{2 + \sqrt{12}}{-2} = \underline{\underline{-1 - \sqrt{3}}}$$
$$x_2 = \frac{2 - \sqrt{12}}{-2} = \underline{\underline{-1 + \sqrt{3}}}$$

$$x_1 = -1 - 2\sqrt{3} \Rightarrow y = -(-1 - \sqrt{3})^2 - 3 \cdot (-1 - \sqrt{3}) + 4$$
$$y = -1 - 2\sqrt{3} - 3 + 3 + 3\sqrt{3} + 4$$
$$\underline{\underline{y = 3 + \sqrt{3}}}$$

$$x_2 = -1 + 2\sqrt{3} \Rightarrow y = -(-1 + \sqrt{3})^2 - 3 \cdot (-1 + \sqrt{3}) + 4$$
$$y = -1 + 2\sqrt{3} - 3 + 3 - 3\sqrt{3} + 4$$
$$\underline{\underline{y = 3 - \sqrt{3}}}$$

$$R : \underline{\underline{P_1(-1 - \sqrt{3}, 3 + \sqrt{3})}} \ \underline{\underline{P_2(-1 + \sqrt{3}, 3 - \sqrt{3})}}$$

$y = -x^2 - 3x + 4$

$D = b^2 - 4ac$
$D = 9 - 4 \cdot (-1) \cdot (4)$
$D = 25$

1) $T(p, q)$
2) $x_1 = ?\ \ x_2 = ?$

$y = -x + 2$

x	y
0	2
1	1

$$p = \frac{-b}{2a}$$

$$p = \frac{3}{-2} = \underline{\underline{-\frac{3}{2}}}$$

$$q = \frac{-D}{4a}$$

$$q = \frac{-25}{-4} = \underline{\underline{\frac{25}{4}}}$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

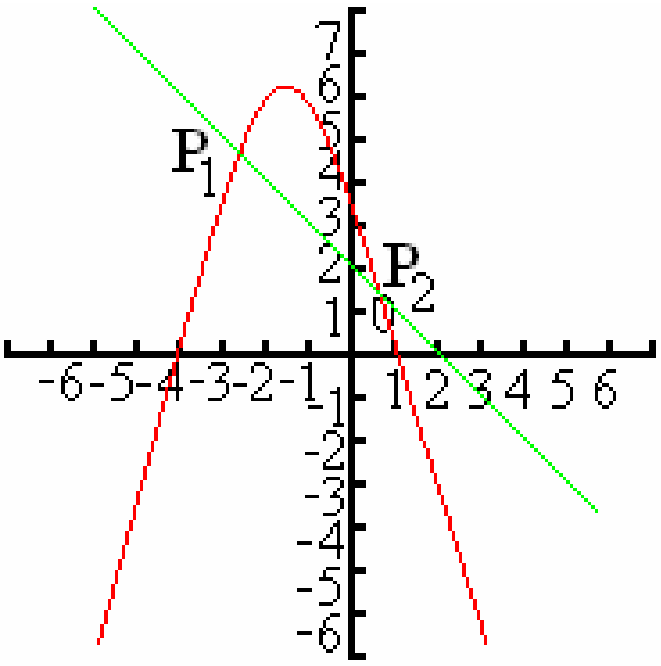
$$x_1 = \frac{3+\sqrt{25}}{-2} = \frac{3+5}{-2} = \frac{8}{-2} = \underline{\underline{-4}}$$

$$x_2 = \frac{3-\sqrt{25}}{-2} = \frac{3-5}{-2} = \frac{-2}{-2} = \underline{\underline{1}}$$

$a < 0 \ \cap$

$$\underline{\underline{1)T(-\frac{3}{2}, \frac{25}{4})}}$$

$$\underline{\underline{2)x_1 = -4 \quad x_2 = 1}}$$



14.) Določi neznan parameter, da bo premica tangenta na parabolo.
(a)

$$y = -x + n$$

$$f(x) = -6x^2 - x + 12$$

$$-x + n = -6x^2 - x + 12$$

$$0 = -6x^2 + 12 - n, \ D = 0$$

$$D = b^2 - 4ac$$

$$0 = 0 - 4 \cdot (-6) \cdot (12 - n)$$

$$0 = 288 - 24n$$

$$\underline{\underline{n = 12}} \Rightarrow D(0,12)$$

(b)

$$y = 3x + n$$
$$f(x) = -2x^2 - x + 10$$

$$3x + n = -2x^2 - x + 10$$
$$0 = -2x^2 - 4x - n + 10,\ D = 0$$

$$D = b^2 - 4ac$$
$$0 = 16 - 4 \cdot (-2) \cdot (10 - n) =$$
$$0 = 16 + 80 - 8n$$
$$8n = 96$$

$$\underline{\underline{n = 12}}$$

$$0 = -2x^2 - 4x - n + 10$$
$$0 = -2x^2 - 4x - 12 + 10$$
$$0 = -2x^2 - 4x - 2$$
$$0 = -2 \cdot (x + 1) \cdot (x + 1)$$

$$\underline{\underline{x = -1}}$$

$$y = 3x + n$$
$$y = 3 \cdot (-1) + 12$$
$$y = -3 + 12$$

$$\underline{\underline{y = 9}}$$

$$\underline{\underline{D(-1,9)}}$$

(c)

$$y = kx - 4$$
$$f(x) = x^2 - 3x - 4$$

$$kx - 4 = x^2 - 3x - 4$$
$$0 = x^2 - 3x - kx\,,\ D = 0$$
$$0 = x^2 x \cdot (3 + k)$$

$$D = b^2 - 4ac$$
$$0 = (3 + k)^2 - 4 \cdot (1) \cdot (0)$$
$$0 = k^2 + 6k + 9$$
$$0 = (k + 3) \cdot (x + 3)$$

$$\underline{\underline{k_1 = -3}}$$
$$\underline{\underline{k_2 = -3}}$$

$$0 = x^2 x \cdot (3 + k)$$
$$0 = x^2 + x \cdot (3 - 3)$$
$$0 = x^2$$

$$\underline{\underline{x = 0}}$$

$$y = kx - 4$$
$$y = -3 \cdot 0 - 4$$

$$\underline{\underline{y = -4}}$$

$$\underline{\underline{D(0,-4)}}$$

(d)

$$y = kx - 14$$
$$y = x^2 - 9x - 10$$

$$kx - 14 = x^2 - 9x - 10$$
$$0 = x^2 - 9x - kx + 4\,,\ D = 0$$
$$0 = x^2 - x(9 + k) + 4$$

$$D = b^2 - 4ac$$
$$0 = (9 + k)^2 - 4 \cdot (1) \cdot (4)$$
$$0 = k^2 + 18k + 81 - 16$$
$$0 = k^2 + 18k + 65$$
$$0 = (k + 13) \cdot (x + 5)$$

$$\underline{\underline{k_1 = -13}}\ \ \underline{\underline{k_2 = -5}}$$

$$0 = x^2 - x(9 + k) + 4$$
$$0 = x^2 - x(9 - 13) + 4$$
$$0 = x^2 + 4x + 4$$
$$0 = (x + 2) \cdot (x + 2)$$

$$\underline{\underline{x_1 = -2}}\ \ \underline{\underline{x_2 = -2}}$$

$$y_1 = kx - 14$$
$$y_1 = -13 \cdot (-2) - 14$$
$$y_1 = 26 - 14$$

$$\underline{\underline{y_1 = 12}}$$

$$0 = x^2 - x(9 - 5) + 4$$
$$0 = x^2 - 4x + 4$$
$$0 = (x - 2) \cdot (x - 2)$$

$$\underline{\underline{x_1 = 2}}\ \ \underline{\underline{x_2 = 2}}$$

$$y_2 = (-5)\cdot 2 - 14$$
$$y_2 = -10 - 14$$

$$\underline{\underline{y_2 = -24}}$$

$$\underline{\underline{D_1(-2,12)}}\ \ \underline{\underline{D_2(2,-24)}}$$

15.) Določi *m*, da bo imel sistem natanko eno rešitev.
(a)

$$x^2 + mx + 1 = 0$$
$$x^2 + x + m = 0$$

$$x^2 + mx + 1 = x^2 + x + m$$
$$mx - x - m + 1 = 0$$
$$x(m-1) - m + 1 = 0$$

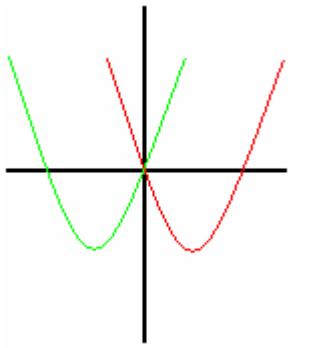
$$\underline{\underline{m \in \Re / \{1\}}}$$

(b)

$$x^2 + 2x - 3m - 5 = 0$$
$$x^2 - 3x - m = 0$$

$$x^2 + 2x - 3m - 5 = x^2 - 3x - m$$
$$5x - 2m + 5 = 0$$

$$\underline{\underline{m \in \Re}}$$



16.) Trikotnikove stranice merijo 26, 54, 90 cm. Če jih podaljšamo za isto dolžino dobimo pravokotni trikotnik. Določi dimenzije novega trikotnika.

$$(90+x)^2 = (26+x)^2 + (54+x)^2$$
$$8100 + 180x + x^2 = 676 + 52x + x^2 + 2196 + 108x + x^2$$
$$0 = x^2 - 20x - 4508$$

$$D = b^2 - 4ac$$
$$D = 400 - 4 \cdot (1) \cdot (-4508)$$
$$D = 18432$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$
$$x_1 = \frac{20 + \sqrt{18432}}{2} = \frac{20 + 96\sqrt{2}}{2} = \underline{\underline{10 + 48\sqrt{2}}}$$
$$x_2 = \frac{20 - \sqrt{18432}}{2} = \frac{20 - 96\sqrt{2}}{2} = 10 - 48\sqrt{2}$$

$$\underline{\underline{a_1 = 36 + 48\sqrt{2}}}$$
$$\underline{\underline{b_2 = 64 + 48\sqrt{2}}}$$
$$\underline{\underline{c_1 = 100 + 48\sqrt{2}}}$$

17.) Vsota števk dvomestnega števila je 8, vsota kvadratov teh števk pa je za 1 manjša od samega števila. Določi to število.

x....desetice
y....enice
$\uparrow$
$10x + y$

$$x + y = 8$$
$$y = 8 - x$$
$$x^2 - y^2 = 10x - y - 1$$

$$x^2+(8-x)^2=10x+(8-x)-1$$
$$x^2+64-16x+x^2=10x+8-x-1$$
$$2x^2-25x+57=0$$

$$D=b^2-4ac$$
$$D=625-4\cdot(2)\cdot(57)$$
$$D=625-456$$
$$D=169$$

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$$
$$x_1=\frac{25+\sqrt{169}}{4}=\frac{25+13}{4}=\frac{19}{2}$$
$$x_2=\frac{25-\sqrt{169}}{4}=\frac{25-13}{4}=\underline{\underline{3}}$$

$$y=8-x=8-3=\underline{\underline{5}}$$

$$\underline{\underline{x=3}}\ \underline{\underline{y=5}}$$
$$\uparrow$$
Iskano število 35.

18.) Vsota kvadratov treh zaporednih lihih števil je 875. Izračunaj ta števila.

$$1.št....2x+1$$
$$2.št....2x+3$$
$$3.št....2x+5$$

$$(2x+1)^2+(2x+3)^2+(2x+5)^2=875$$
$$4x^2+2x+1+4x^2+12x+9+4x^2+20x+25=875$$
$$12x^2+36x-840=0$$

$$D=b^2-4ac$$
$$D=1296-4\cdot(12)\cdot(-840)$$
$$D=41616$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{-36 + \sqrt{41616}}{24} = \frac{-36 + 204}{24} = \frac{168}{24} = \underline{\underline{7}}$$

$$x_2 = \frac{-36 - \sqrt{41616}}{24} = \frac{-36 - 204}{24} = -\frac{240}{24} = \underline{\underline{-10}}$$

1.)7

$$(2 \cdot 7 + 1)^2 + (2 \cdot 7 + 3)^2 + (2 \cdot 7 + 5)^2 = 875$$

$$15^2 + 17^2 + 19^2 = 875$$

2.)−10

$$(2 \cdot (-10) + 1)^2 + (2 \cdot (-10) + 3)^2 + (2 \cdot (-10) + 5)^2 = 875$$

$$(-19)^2 + (-17)^2 + (-15)^2 = 875$$

Ta števila so 15,17 19 ali −15,−17,−19.

19.) Ploščina pravokotnika, katerega dolžine so tri zaporedna cela števila meri 96 cm^2. Določi obseg.

!?!

20.) Reši neenačbo:

(a)

$$x^2 - 16 \leq 0$$

$$x^2 - 16 = 0$$

$$(x - 4) \cdot (x + 4) = 0$$

$$R : \underline{\underline{-4 \leq x \leq 4}}$$

(b)

$$-x^2 + 2x + 3 < 0$$

$$-x^2 + 2x + 3 = 0$$

$$-(x - 3) \cdot (x + 1) = 0$$

$$x_1 = 3 \quad x_2 = -1$$

$$R : \underline{\underline{(x < -1) \vee (x > 3)}}$$

(c)

$$2x^2 - x - 10 < 0$$

$$2x^2 - x - 10 = 0$$

$$x^2 - \frac{1}{2}x - 5 = 0$$

$$(x + 2) \cdot (x - \frac{5}{2}) < 0$$

$$x_1 = -2 \quad x_2 = \frac{5}{2}$$

$$\underline{\underline{R : (-2 < x < \frac{5}{2})}}$$

(d)

$$-7x^2 + x - 13 > 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$x_1 = \frac{-1 - \sqrt{b^2 - 4ac}}{2 \cdot (-7)} =$$

$$= \frac{-1 - \sqrt{1^2 - 4 \cdot (-7) \cdot (-13)}}{-14} =$$

$$= \frac{-1 - \sqrt{-363}}{-14} = \underline{\underline{\frac{-1 - \sqrt{363}i}{-14}}}$$

$$x_2 = \underline{\underline{\frac{-1 + \sqrt{363}i}{-14}}}$$

$$\underline{\underline{R : x \notin \Re}}$$

21.) Reši sisteme neenačb:

(a)

$$(x^2 - 11x + 24 \le 0) \wedge (-x^2 + 5x + 14 \ge 0)$$

$$x^2 - 11x + 24 \le 0$$
$$(x - 8) \cdot (x - 3) = 0$$
$$\underline{\underline{x_1 = 8}} \quad \underline{\underline{x_2 = 3}}$$

$$-x^2 + 5x + 14 \ge 0 \,/\cdot(-1)$$
$$x^2 - 5x - 14 \le 0$$
$$(x - 7) \cdot (x + 2) = 0$$
$$\underline{\underline{x_1 = 7}} \quad \underline{\underline{x_2 = -2}}$$

$$\underline{\underline{R : \{3 \le x \le 7\}}}$$

(b)

$$(x^2 - 3x - 4 < 0) \wedge (x^2 - 4x - 21 > 0)$$

$$x^2 - 3x - 4 < 0$$
$$(x + 1) \cdot (x - 4) = 0$$
$$\underline{\underline{x_1 = -1}} \quad \underline{\underline{x_2 = 4}}$$

$$x^2 - 4x - 21 > 0$$
$$(x - 7) \cdot (x + 3) = 0$$
$$\underline{\underline{x_1 = 7}} \quad \underline{\underline{x_2 = -3}}$$

$$\underline{\underline{R : \{\varnothing\}}}$$

(c)

$$(2x^2 + 5x + 2 < 0) \wedge (-x^2 + 3x + 10 > 0)$$

$$2x^2 + 5x + 2 < 0$$
$$D = b^2 - 4ac = 25 - 4 \cdot 2 \cdot 2 = \sqrt{9} = \underline{\underline{3}}$$
$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-5 + 3}{4} = \underline{\underline{-\frac{1}{2}}}$$
$$x_2 = \frac{-5 - 3}{4} = \frac{-8}{4} = \underline{\underline{-2}}$$

$$-x^2+3x+10>0\,/\cdot(-1)$$
$$x^2-3x-10<0$$
$$(x+2)\cdot(x-5)<0$$
$$\underline{\underline{x_1=-2}}\quad\underline{\underline{x_2=5}}$$

$$\underline{\underline{R: x\in(-2,-\frac{1}{2})}}$$

(d)

$$6x<x^2<4$$

$$6x<x^2$$
$$6x-x^2<0$$
$$(6-x)x<0$$
$$\underline{\underline{x_1=6}}\quad\underline{\underline{x_2=0}}$$

$$x^2<4$$
$$x^2-4<0$$
$$(x+2)\cdot(x-2)<0$$
$$\underline{\underline{x_1=-2}}\quad\underline{\underline{x_2=2}}$$

$$\underline{\underline{R:\{-2<x<0\}}}$$

(e)

$$0<x^2-x<12$$

$$0<x^2-x$$
$$0<x(x-1)$$
$$\underline{\underline{x_1=1}}\quad\underline{\underline{x_2=0}}$$

$$x^2-x<12$$
$$x^2-x-12<0$$
$$(x-4)\cdot(x+3)<0$$
$$\underline{\underline{x_1=4}}\quad\underline{\underline{x_2=-3}}$$

$$\underline{\underline{R:\{(-3<x<0)\vee(1<x<4)\}}}$$

22.) Reši neenačbo:

(a)

$$\frac{x^2+6x+9}{x^2+1} \le 0$$

$$1.)(x^2+6x+9 \le 0) \wedge (x^2+1>0)$$

$$x^2+6x+9 \le 0$$
$$(x+3)\cdot(x+3)=0$$
$$\underline{\underline{x_1=-3 \quad x_2=-3}}$$

$$x^2+1>0$$
$$\underline{\underline{x \in \Re}}$$

$$2.)(x^2+6x+9 \ge 0) \wedge (x^2+1 \le 0)$$

$$x^2+6x+9 \ge 0$$
$$(x+3)^2 \ge 0$$
$$\underline{\underline{x \in \Re}}$$

$$x^2+1 \le 0$$
$$x^2 \le -1$$
$$\underline{\underline{x=\{\varnothing\}}}$$

$$\underline{\underline{R: x=-3}}$$

(b)

$$\frac{x^2+5x}{x^2-4x-21} \le 0$$

$$1.)(x^2+5x \le 0) \wedge (x^2-4x-21 \ge 0)$$

$$x^2-5x \le 0$$
$$x(x+5) \le 0$$
$$\underline{\underline{x \in [-5,0]}}$$

$$x^2-4x-21 \ge 0$$
$$(x-7)\cdot(x+3) \ge 0$$
$$\underline{\underline{x \in (-8,3] \cup [7,+\infty)}}$$

$2.)(x^2+5x\geq 0)\wedge(x^2-4x-21\leq 0)$

$x^2-5x\geq 0$
$x(x-5)\geq 0$
$\underline{\underline{x\in(-\infty,-5]\cup[0,+\infty)}}$

$x^2-4x-21\leq 0$
$(x-7)\cdot(+3)\leq 0$
$\underline{\underline{x\in[-3,7]}}$

$\underline{\underline{R: x\in[-5,-3]\cup[0,7]}}$

(c)

$$\frac{x^2-3x+4}{1-x^2}>0$$

$1.)(x^2-3x+4>0)\wedge(1-x^2<0)$

$x^2-3x+4>0$
$D=b^2-4ac=9-4\cdot 1\cdot 4=\underline{\underline{-7}}$
$\underline{\underline{x\in\{\varnothing\}}}$

$1-x^2<0$
$x^2>1$
$\underline{\underline{x\in(-\infty,-1)\cup(1,+\infty)}}$

$\underline{\underline{x\in\{\varnothing\}}}$

$2.)(x^2-3x+4>0)\wedge(1-x^2>0)$

$x^2-3x+4>0$
$\underline{\underline{x\in\mathfrak{R}}}$

$1-x^2>0$
$x^2<1$
$\underline{\underline{x\in\{\varnothing\}}}$

$\underline{\underline{R: x\in\{\varnothing\}}}$

23.) Določi parameter *a*, da bosta rešitvi enačbe realni.

$$2x^2 + (a-9)x + a^2 + 3a + 4 = 0,\ D \geq 0$$

$$D = b^2 - 4ac$$
$$(a-9)^2 - 4 \cdot 2 \cdot (a^2 + 3a + 4) \geq 0$$
$$a^2 - 18a + 81 - 8a^2 - 24a - 32 \geq 0$$
$$-7a^2 - 42a + 49 \geq 0$$

$$D = b^2 - 4ac$$
$$D = 1764 - 4 \cdot (-7) \cdot (49) = \sqrt{3136} = \underline{\underline{56}}$$

$$a_1 = \frac{-b + \sqrt{D}}{2a} = \frac{42 + 56}{-14} = \frac{98}{-14} = \underline{\underline{-7}}$$
$$a_2 = \frac{42 - 56}{-14} = \frac{-14}{-14} = \underline{\underline{1}}$$

$$\underline{\underline{R : (-7 \leq a \leq 1)}}$$

24.) Določi vrednost parametra *m*, da bo funkcija pozitivna za vsak x.

$$f(x) = (m^2 - 1)x^2 + 2(m-1)x + 2,\ D < 0,\ a > 0,\ m^2 - 1 > 0,\ m^2 > 1$$

$$D = b^2 - 4ac$$
$$(2m-2)^2 - 4 \cdot (m^2 - 1) \cdot (2) < 0$$
$$4m^2 - 8m + 4 - 8m^2 + 8 < 0$$
$$-4m^2 - 8m + 12 < 0 \,/\, :4$$
$$-m^2 - 2m + 3 < 0 \,/ \cdot (-1)$$
$$m^2 + 2m - 3 > 0$$
$$(m-1) \cdot (m+3) > 0$$

$$\underline{\underline{m \in (-\infty, -3) \cup (1, +\infty)}}$$

25.) Določi vrednost parametra *a*, da bo neenačba izpolnjena za vsak realni x.

$$ax^2 - (a+2)x + a + 2 > 0 \quad D < 0,\ a > 0$$

$$D = b^2 - 4ac$$
$$0 > b^2 - 4ac$$
$$0 > a^2 + 4a + 4 - 4a^2 - 8a$$
$$0 > -3a^2 - 4a + 4 \,/\cdot(-1)$$
$$0 < 3a^2 + 4a - 4$$
$$0 < (3a-2)\cdot(a+2)$$
$$0 < (a - \frac{2}{3})\cdot(a+2)$$

$$\underline{\underline{a > \frac{2}{3}}}$$