Potence, koreni, kotne funkcije ostrih kotov

1.

$$3^{143}-4\cdot 3^{141}=3^{141}(3^2-4)=3^{140}\cdot 15=5\cdot\underbrace{3^{141}}_{k_1}=15\cdot\underbrace{3^{140}}_{k_2}$$

Število je deljivo z 5 in 15.

2. (a)

$$\begin{aligned} a^{(x-1)^2}\cdot(a^{x+2})^{x-2}:(a^{x-2})^{2x} &= a^{x^2-2x+1}\cdot a^{x^2-4}:a^{2x^2-4x}= \\ &= a^{x^2-2x+1+x^2-4}:a^{2x^2-4x} \\ &= a^{2x^2-2x-3-2x^2+4x} \\ &= a^{2x-3} \end{aligned}$$

(b)

$$\begin{aligned} \sqrt[4]{x^3}\cdot\sqrt[12]{x^{-2}y\sqrt{xy^{-2}}}:(\sqrt{x}\cdot\sqrt[6]{y^2}\cdot\sqrt[24]{x^{-3}}) &= \sqrt[24]{x^{18}}\cdot\sqrt[24]{x^{-4}y^2xy^{-2}}:(\sqrt[24]{x^{12}}\cdot\sqrt[24]{y^8}\cdot\sqrt[24]{x^{-3}} \\ &= \sqrt[24]{x^{18}x^{-3}}:\sqrt[24]{x^{12}y^8x^{-3}}=\sqrt[24]{\frac{x^{15}}{x^9y^8}}=\sqrt[24]{\frac{x^6}{y^8}}=\frac{\sqrt[4]{x}}{\sqrt[3]{x}} \end{aligned}$$

(c)

$$\sqrt[4]{x^3\sqrt[3]{xy\sqrt{xy^{-2}}}}=\sqrt[24]{x^{18}x^2y^2xy^{-2}}=\sqrt[24]{x^{21}}=\sqrt[8]{x^7}$$

(d)

$$\begin{aligned} \sqrt{x\sqrt[3]{8y\cdot\sqrt{z}}}:\sqrt[4]{2x^3\cdot\sqrt[3]{y^2z}} &= \sqrt[12]{x^6 64y^2z}:\sqrt[12]{8x^9y^2z} \\ &= \frac{\sqrt[12]{x^6 64y^2z}}{\sqrt[12]{8x^9y^2z}}=\frac{\sqrt[4]{2}}{\sqrt[4]{x}} \end{aligned}$$

(e)

$$\begin{aligned} \frac{2-x}{x^3-x^2-x+1}:(\frac{1}{x-1}\cdot\frac{x}{x+1}-\frac{2}{x+1}) &= \frac{2-x}{x^3-x^2-x+1}:\frac{x-(2x-2)}{x^2-1} \\ &= \frac{(2-x)\cdot(x^2-1)}{(x^3-x^2-x+1)\cdot(-x+2)} \\ &= \frac{(x^2-1)}{(x^3-x^2-x+1)}=\frac{(x^2-1)}{x^2(x-1)-(x-1)} \\ &= \frac{(x^2-1)}{(x-1)\cdot(x^2-1)}=\frac{1}{x-1} \end{aligned}$$

(f)

$$\begin{aligned} \frac{1}{8}\cdot 4^{n+3}-12\cdot 4^{n-1}-\frac{1}{2}4^{n+1} &= \frac{1}{8}4^{n-1}(4^4-96-4^3) \\ &= \frac{1}{8}4^{n-1}(96)=\frac{96}{8}4^{n-1}=12\cdot 4^{n-1} \\ &= 3\cdot 4\cdot 4^{n-1}=3\cdot 4^n \end{aligned}$$

(g)

$$\begin{aligned} \frac{2^x-2^{x+1}-2^{x-2}}{2^{x+2}}-\frac{4^{5-x}-4^{4-x}-4^{3-x}}{4^{5-x}}+\frac{8^{x+1}}{2^x\cdot 4^{x+1}} &= \frac{2^{x-2}(2^2-2^3-1)}{2^{x+2}}-\frac{4^{3-x}(4^2-4-1)}{4^{5-x}}+\frac{8^{x+1}}{2^x\cdot 4^{x+1}} \\ &= \frac{2^{x-2}(-5)}{2^{x+2}}-\frac{4^{3-x}(11)}{4^{5-x}}+\frac{2^{3x+3}}{2^x\cdot 2^{2x}2^2} \\ &= \frac{-5}{2^4}-\frac{11}{4^2}+\frac{2^{3x+3}}{2^{3x+2}}=\frac{-5}{16}-\frac{11}{16}+\frac{2}{1}=-\frac{16}{16}+2=1 \end{aligned}$$

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(h)

$$\begin{aligned}\frac{3\cdot 2^{2x-1}-9\cdot 2^{2x}+5\cdot 2^{2x-2}}{9\cdot 4^{x}+4^{x+2}} &= \frac{2^{2x-2}(3\cdot 2-9\cdot 2^{2}+5)}{4^{x}(9+4^{2})}=\frac{2^{2x-2}(-25)}{2^{2x}(25)}\\ &= -\frac{2^{-2}}{1}=-\frac{1}{4}\end{aligned}$$

(i)

$$\begin{aligned}\left(\frac{3x^{-\frac{1}{3}}}{x^{\frac{2}{3}}-2x^{-\frac{1}{3}}}-\frac{x^{\frac{1}{3}}}{x^{\frac{4}{3}}-x^{\frac{1}{3}}}\right)^{-1} &= \left(\frac{3x^{-\frac{1}{3}}}{x^{-\frac{1}{3}}(x-2)}-\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}(x-1)}\right)^{-1}\\ &= \left(\frac{3}{x-2}-\frac{1}{x-1}\right)^{-1}\\ &= \left(\frac{3(x-1)}{(x-2)(x-1)}-\frac{x-2}{(x-1)(x-2)}\right)^{-1}\\ &= \left(\frac{3x-3-x+2}{(x-2)(x-1)}\right)^{-1}=\left(\frac{2x-1}{(x-2)(x-1)}\right)^{-1}\\ &= \frac{(x-2)(x-1)}{2x-1}\end{aligned}$$

(j)

$$\begin{aligned}\left(\frac{a}{a^{2}+a-12}+\frac{1}{a^{2}+5a+4}\right)\cdot\left(1-\frac{4a}{a^{2}+2a-3}\right) &= \left(\frac{a}{(a+4)(a-3)}+\frac{1}{(a+4)(a+1)}\right)\cdot\left(1-\frac{4a}{(a+3)(a-1)}\right)\\ &= \left(\frac{a(a+1)+(a-3)}{(a+4)(a-3)(a+1)}\right)\cdot\left(\frac{(a+3)(a-1)-4a}{(a+3)(a-1)}\right)\\ &= \left(\frac{a^{2}+2a-3}{(a+4)\,(a-3)\,(a+1)}\right)\cdot\left(\frac{a^{2}-2a-3}{(a+3)\,(a-1)}\right)\\ &= \frac{(a+3)\,(a-1)\,(a-3)\,(a+1)}{(a+4)\,(a-3)\,(a+1)\,(a+3)\,(a-1)}=\frac{1}{a+4}\end{aligned}$$

(k)

$$\begin{aligned}\frac{3a+15}{a^{n+1}-2a^{n}}-\frac{3a^{1-n}-5a^{-n}}{a+2} &+ \frac{a^{2}-35a-30}{a^{n+2}-4a^{n}}=\frac{(3a+15)}{a^{n}(a-2)}-\frac{3a^{1-n}-5a^{-n}}{a+2}+\frac{a^{2}-35a-30}{a^{n}(a^{2}-4)}\\ &= \frac{(3a+15)(a+2)-(3a^{1-n}-5a^{-n})(a^{n+1}-2a^{n})+a^{2}-35a-30}{a^{n}(a-2)(a+2)}\\ &= \left(\frac{3a^{2}+21a+30-(3a^{2}-6a-5a+10)+a^{2}-35a-30}{a^{n}(a-2)(a+2)}\right)\\ &= \left(\frac{3a^{2}+21a+30-3a^{2}+6a+5a-10+a^{2}-35a-30}{a^{n}(a-2)(a+2)}\right)\\ &= \left(\frac{a^{2}-3a-10}{a^{n}(a-2)(a+2)}\right)=\left(\frac{(a-5)(a+2)}{a^{n}(a-2)(a+2)}\right)=\frac{(a-5)}{a^{n}(a-2)}\end{aligned}$$

3. (a)

$$\sqrt{28900}:\sqrt{0,16}=\frac{170}{0,4}=425$$

(b)

$$125^{-\frac{1}{3}}\cdot 27^{\frac{2}{3}}:9^{\frac{1}{2}}=\sqrt[3]{125^{-1}}\cdot\sqrt[3]{27^{2}}\cdot\sqrt{9}=\frac{1}{5}\cdot 9:3=\frac{3}{5}$$

(c)

$$\begin{aligned}\sqrt{3-\sqrt{5}}(3+\sqrt{5})(\sqrt{10}-\sqrt{2})= &= \sqrt{(3-\sqrt{5})(3+\sqrt{5})}(\sqrt{3+\sqrt{5}})(\sqrt{10}-\sqrt{2})\\ &= \sqrt{9-5}\sqrt{(3+\sqrt{5})(\sqrt{10}-\sqrt{2})^{2}}\\ &= 2\sqrt{(3+\sqrt{5})(10-2\sqrt{20}+2)}\\ &= 2\sqrt{(3+\sqrt{5})(12-2\sqrt{20})}\\ &= 2\sqrt{(3+\sqrt{5})(3-\sqrt{5})\cdot 4}\\ &= 2\sqrt{(9-5)4}=2\sqrt{16}=8\end{aligned}$$

(d)

$$\begin{aligned}\frac{\sqrt{2}+1}{\sqrt{2}-1}-2\sqrt{4+\sqrt{14}}\cdot 2\sqrt{4-\sqrt{14}} &= \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}-2\sqrt{16-14}\\ &= \frac{3+2\sqrt{2}}{2-1}-2\sqrt{2}=3\end{aligned}$$

(e)

$$\begin{aligned}(16^{\frac{1}{8}}+(27^{-\frac{2}{3}})^{-\frac{1}{2}})(2^{0,5}-(\frac{1}{9})^{-0,5}) &+ \frac{1}{\sqrt{6}-1}\cdot\sqrt{7-2\sqrt{6}}(1+\sqrt{6})\\ &= (2^{\frac{1}{2}}+3)(2^{\frac{1}{2}}-3)+\frac{\sqrt{7-2\sqrt{6}}(1+\sqrt{6})}{\sqrt{6}-1}\\ &= 2-9+\frac{\sqrt{7-2\sqrt{6}(1+\sqrt{6})^2}}{(\sqrt{6}-1)}\\ &= -7+\frac{\sqrt{(7-2\sqrt{6})(7+2\sqrt{6})}}{(\sqrt{6}-1)}=-7\frac{\sqrt{49-24}}{(\sqrt{6}-1)}\\ &= -7+\frac{5}{(\sqrt{6}-1)}=--7+\frac{5(\sqrt{6}+1)}{(\sqrt{6}-1)(\sqrt{6}+1)}\\ &= -7+\frac{5(\sqrt{6}+1)}{5}=-7+(\sqrt{6}+1)=-6+\sqrt{6}\end{aligned}$$

(f)

$$\begin{aligned}((16^{\frac{3}{4}})^{-\frac{2}{3}}\cdot(27^{\frac{1}{3}})^{-2})^{-\frac{1}{2}}+\sqrt{32^{-0,2}-((-3-\frac{3}{8})^{-\frac{1}{3}})^{-1}} &= =(16^{-\frac{1}{2}}\cdot 27^{-\frac{2}{3}})^{-\frac{1}{2}}+\sqrt{\frac{1}{2}-(-\frac{27}{8})^{\frac{1}{3}}}\\ &= (16^{\frac{1}{4}}\cdot 9^{\frac{3}{6}})+\sqrt{\frac{1}{2}+\frac{3}{2}}\\ &= (2\cdot 3)+\sqrt{\frac{1}{2}+\frac{3}{2}}=6+\sqrt{\frac{4}{2}}=6+\sqrt{2}\end{aligned}$$

(g)

$$2\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}}=2\sqrt[64]{2^{32}\cdot 2^{16}\cdot 2^{8}\cdot 2^{4}\cdot 2^{2}\cdot 2}=2\sqrt[64]{2^{63}}$$

(h)

$$\begin{aligned}\sqrt{(2,5)^3\cdot 0,8^3\cdot(\frac{9}{5})^{-2}\cdot 0,75^4\cdot(\frac{2}{3})^5} &= \sqrt{\left(\frac{5}{2}\cdot\frac{4}{5}\right)^3\cdot\frac{5^2}{3^4}\cdot\frac{3^4}{2^8}\cdot\frac{2^5}{3^5}}\\ &= \sqrt{\frac{5^3\cdot 2^6\cdot 5^2\cdot 3^4\cdot 2^5}{2^3\cdot 5^3\cdot 3^4\cdot 2^8\cdot 3^5}}\\ &= \sqrt{\frac{25}{32}}=\frac{5}{4\sqrt{2}}=\frac{5\sqrt{2}}{16}\end{aligned}$$

4. (a)

$$\frac{1}{\sqrt[3]{x}}=\frac{\sqrt[3]{x^2}}{\sqrt[3]{x}\cdot\sqrt[3]{x^2}}=\frac{\sqrt[3]{x^2}}{x}$$

(b)

$$\frac{\sqrt{15}}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{15}\cdot(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}=\frac{\sqrt{75}+\sqrt{45}}{2}=\frac{5\sqrt{3}+\sqrt{15}\cdot\sqrt{3}}{2}=\frac{\sqrt{3}(5+\sqrt{15})}{2}$$

(c)

$$\frac{2\sqrt{3}}{\sqrt{2}-1}=\frac{(2\sqrt{3})(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{2\sqrt{6}-2\sqrt{3}}{2-1}=2\sqrt{3}(\sqrt{2}-1)$$

(d)

$$\begin{aligned}
\frac{\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}} &= \frac{(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})^2}{(\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}})(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})}\\
&= \frac{(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})^2}{(2+\sqrt{3})-(2-\sqrt{3})}\\
&= \frac{(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})^2\sqrt{3}}{2\sqrt{3}(\sqrt{3})}\\
&= \frac{(2+\sqrt{3}+2(\sqrt{2+\sqrt{3}})(\sqrt{2-\sqrt{3}})+2-\sqrt{3})(\sqrt{3})}{6}\\
&= \frac{(4+2\sqrt{(2+\sqrt{3})(2-\sqrt{3})})\sqrt{3}}{6} = \frac{(4+2(\sqrt{4-3})\sqrt{3}}{6} = \frac{6\sqrt{3}}{6} = \sqrt{3}
\end{aligned}$$

(e)

$$\begin{aligned}
\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}-\sqrt{5}} = \frac{2\sqrt{6}\cdot((\sqrt{2}+\sqrt{3})+\sqrt{5})}{((\sqrt{2}+\sqrt{3})-\sqrt{5})((\sqrt{2}+\sqrt{3})+\sqrt{5})} &= \frac{2\sqrt{12}+2\sqrt{18}+2\sqrt{30}}{2+2\sqrt{6}+3-5}\\
&= \frac{2\sqrt{6}(\sqrt{2}+\sqrt{3}+\sqrt{5})}{2\sqrt{6}} = \sqrt{2}+\sqrt{3}+\sqrt{5}
\end{aligned}$$

5.

$$\begin{aligned}
\left(\frac{\sqrt{3}+2}{\sqrt{3}+1}-\frac{1}{\sqrt{3}+3}\right)\cdot\left(\frac{\sqrt{3}+2}{\sqrt{3}+3}-\frac{1}{\sqrt{3}+1}\right)^{-1} &= (\frac{(\sqrt{3}+2)(\sqrt{3}+3)-(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}+3)})\cdot(\frac{(\sqrt{3}+2)(\sqrt{3}+1)-(\sqrt{3}+3)}{(\sqrt{3}+1)(\sqrt{3}+3)})^{-1}\\
&= \frac{((\sqrt{3}+2)(\sqrt{3}+3)-(\sqrt{3}+1))(\sqrt{3}+1)(\sqrt{3}+3)}{(\sqrt{3}+1)(\sqrt{3}+3)((\sqrt{3}+2)(\sqrt{3}+1)-(\sqrt{3}+3))}\\
&= \frac{(\sqrt{3}+2)(\sqrt{3}+3)-(\sqrt{3}+1)}{(\sqrt{3}+2)(\sqrt{3}+1)-(\sqrt{3}+3)}\\
&= \frac{3+3\sqrt{3}+2\sqrt{3}+6-\sqrt{3}-1}{3+\sqrt{3}+2\sqrt{3}+2-\sqrt{3}-3}\\
&= \frac{8+4\sqrt{3}}{2+2\sqrt{3}} = \frac{2(4+2\sqrt{3})}{2(1+\sqrt{3})}\\
&= 2\frac{(2+\sqrt{3})}{(1+\sqrt{3})}\\
&= 2\frac{(2+\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}\\
&= 2\frac{(2-2\sqrt{3}+\sqrt{3}-3)}{3-1} = 1+\sqrt{3}
\end{aligned}$$

6. *Enakokraki trapez*:

$a = 28$ cm
$b = 11$ cm
$c = 10$ cm

$2x = a - c$
$2x = 28 - 10$
$x = 9$ cm
$\cos\alpha = \frac{x}{b} = \frac{9}{11} \Rightarrow \alpha = \underline{35,1^\circ}$

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7. *Enakokraki trikotnik*:

$a_p \ldots$ projekcija katete a na hipotenuzo c
$b_p \ldots$ projekcija katete b na hipotenuzo c

$a_p = 12cm$
$b_p = 75cm$
$c = ?$
$\alpha = ?$
$\beta = ?$

$v^2 = a_p \cdot b_p$
$v = \sqrt{12\cdot 75} = 30cm$
$\text{tg}\,\alpha = \frac{v}{a}$
$\alpha = \underline{68,2^\circ}$

$c = a_p + b_p = \underline{87}$ cm
$\beta = \gamma - \alpha = 90^\circ - 68,1^\circ = \underline{21,9^\circ}$

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8. *Enakokraki trikotnik:*

$a = b = 20$ cm $\qquad \cos\alpha = \dfrac{\frac{c}{2}}{a} \qquad \gamma = 180^\circ - 2\alpha$

$c = 28$ cm $\qquad \cos\alpha = \dfrac{14}{20} \qquad \gamma = 180^\circ - 2\cdot 45,6^\circ$

$\alpha = ?$ $\qquad \gamma = 88,8^\circ$

$\gamma = ?$ $\qquad \alpha = 45,6^\circ$

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9. *Trapez:*

$x\ldots$ projekcija kraka d na osnovnico a
$y\ldots$ projekcija kraka b na osnovnico a

$$\sin\alpha = \frac{v}{d} = \frac{4}{\frac{8\sqrt{3}}{3}} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = \underline{60^\circ} \Rightarrow \delta = 180^\circ - \alpha = \underline{120^\circ}$$

$$\sin\beta = \frac{v}{b} = \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \beta = \underline{45^\circ} \Rightarrow \gamma = 180^\circ - \beta = \underline{135^\circ}$$

$$\text{tg }\alpha = \frac{v}{x} \Rightarrow x = \frac{v}{\text{tg }\alpha} = \frac{4}{\text{tg }60^\circ} = \frac{4\sqrt{3}}{3}\text{ cm},\ \text{tg }\beta = \frac{v}{y} \Rightarrow y = \frac{v}{\text{tg }\beta} = \frac{4}{\text{tg }45^\circ} = 4\text{ cm},$$

$$a = x + y + c = \frac{4\sqrt{3}}{3} + 4 + 3 = \frac{21 + 4\sqrt{3}}{3}\text{ cm}$$

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10. *Enakokraki trikotnik:*

$|AB| = c = 9$ cm $\qquad \cos\alpha = \frac{\frac{c}{2}}{d} \qquad \sin\alpha = \frac{d}{\frac{c}{2}}$

$|AC| = b|BC| = a = 7,5$ cm $\qquad \cos\alpha = \frac{4,5}{7,5} \qquad d = \sin\alpha\cdot\frac{c}{2}$

$|ED| = d = ?$ $\qquad d = \underline{3,6}$ cm

$\alpha = 53,1^\circ$

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11. a) $$\frac{\sin 45^\circ + \cos 0^\circ}{\sin 0^\circ - \cos 45^\circ} = \frac{\frac{\sqrt{2}}{2} + 1}{0 - \frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}+2}{2}}{-\frac{\sqrt{2}}{2}} = -\frac{\sqrt{2}(\sqrt{2}+1)}{\sqrt{2}} = -\sqrt{2} - 1$$

b) $$\frac{\cos 60^\circ + \sin 30^\circ}{\text{tg }60^\circ + \text{ctg }45^\circ} = \frac{\frac{1}{2} + \frac{1}{2}}{\sqrt{3} + 1} = \frac{1}{1 + \sqrt{3}} = \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{\sqrt{3} - 1}{2}$$

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12. $AC = 15$ cm $\qquad AD^2 = AC^2 - DC^2 \qquad DC^2 = AD\cdot DB \qquad BC^2 = CD^2 + DB^2$

$DC = 12$ cm $\qquad AD^2 = 15^2 - 12^2 \qquad DB = \dfrac{DC^2}{AD} \qquad BC^2 = 12^2 + 16^2$

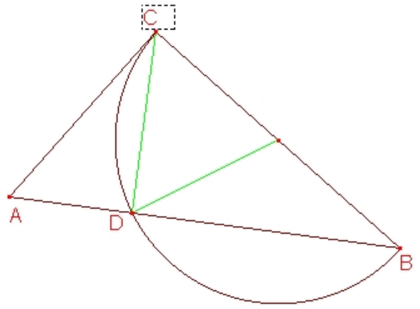
$\text{p}\frac{k}{2} = ?$ $\qquad AD = \sqrt{81} \qquad DB = \dfrac{12^2}{9} \qquad BC = \sqrt{400}$

$\text{o}\frac{k}{2} = ?$ $\qquad AD = 9cm \qquad DB = 16cm \qquad BC = 20cm$

$r = \frac{BC}{2}$ $\qquad p\dfrac{k}{2} = \dfrac{\pi r^2}{2} = \dfrac{\pi 100}{2} = 157cm^2$

$$o\frac{k}{2} = \frac{2\pi r}{2} + 2r = \pi\cdot 10 + 20 = 51,4cm$$

In še: $sin\alpha = \frac{12}{15} \rightsquigarrow \alpha = 53.1^\circ$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