KVADRATNA FUNKCIJA

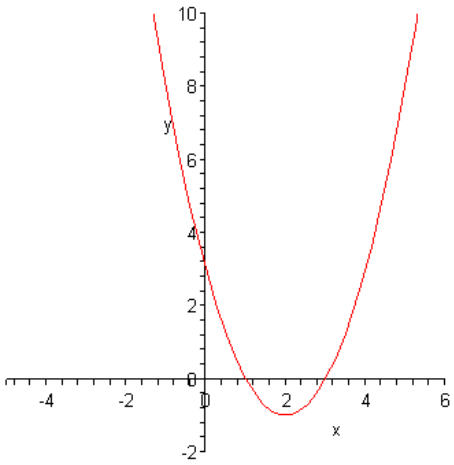
1.

a)

$f(x) = x^2 - 4x + 3$

1. $a > 0$ $\qquad D = b^2 - 4ac = 16 - 12 = 4$
2. $A(0, 3)$
3. $T(p, q) = T(2, -1)$ $\qquad p = \frac{-b}{2a} = \frac{-(-4)}{2} = 2, \qquad q = \frac{-D}{4a} = \frac{-4}{4} = -1$
4. *ničli* :

$x_1 = \frac{-b+\sqrt{D}}{2a} = \frac{4+2}{2} = 3 \qquad x_2 = \frac{-b-\sqrt{D}}{2a} = \frac{4-2}{2} = 1$

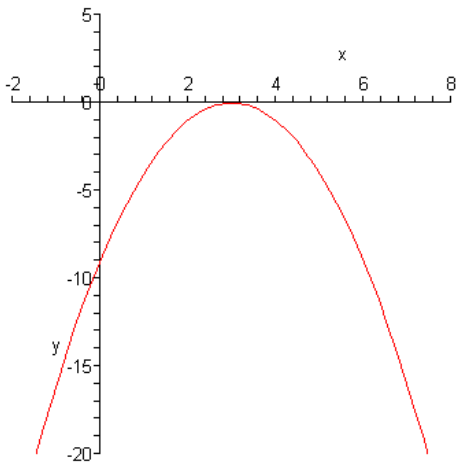


b)

$f(x) = -x^2 + 6x - 9$

1. $a < 0$ $\qquad D = b^2 - 4ac = 36 - 36 = 0$
2. $A(0, -9)$
3. $T(p, q) = T(3, 0)$ $\qquad p = \frac{-b}{2a} = \frac{-6}{-2} = 3, \qquad q = \frac{-D}{4a} = 0$
4. *ničli* :

$D = 0 \rightarrow x_1 = x_2 = \frac{-b}{2a} = 3$

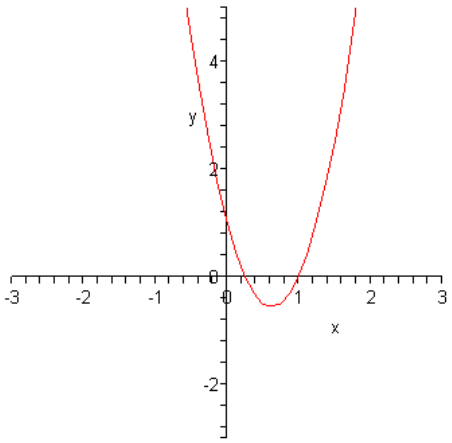


c)

$f(x) = 4x^2 - 5x + 1$

1. $a > 0$
2. $A(0,1)$ $\quad D = b^2 - 4ac = 25 - 16 = 9$
3. $T(p,q) = T(\frac{5}{8}, -\frac{9}{16})$ $\quad p = \frac{-b}{2a} = \frac{5}{8}, \quad q = \frac{-D}{4a} = \frac{-9}{16} = -\frac{9}{16}$
4. *ničli* :

$x_1 = \frac{-b+\sqrt{D}}{2a} = \frac{5+3}{8} = 1, \quad x_2 = \frac{-b-\sqrt{D}}{2a} = \frac{5-3}{8} = \frac{1}{4}$

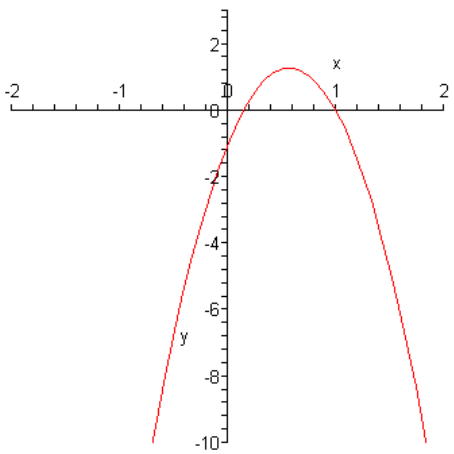


d)

$f(x) = -7x^2 + 8x - 1$

1. $a < 0$
2. $A(0,-1)$ $\quad D = b^2 - 4ac = 8^2 - 28 = 36$
3. $T(p,q) = T(\frac{4}{7}, \frac{9}{7})$ $\quad p = \frac{-b}{2a} = \frac{-8}{-14} = \frac{4}{7}, \quad q = \frac{-D}{4a} = \frac{-36}{-28} = \frac{9}{7}$
4. *ničli* :

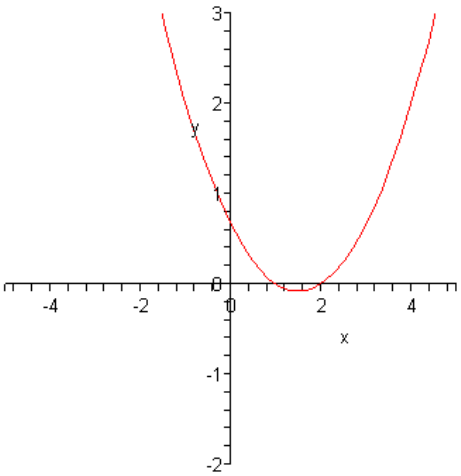
$x_1 = \frac{-b+\sqrt{D}}{2a} = \frac{-8+6}{-14} = \frac{1}{7}, \quad x_2 = \frac{-b-\sqrt{D}}{2a} = \frac{-8-6}{-14} = 1$



$e)$
$f(x) = \frac{x^2}{3} - x + \frac{2}{3}$

1. $a > 0$
2. $A(0, \frac{2}{3})$ $\quad D = b^2 - 4ac = 1 - \frac{8}{9} = \frac{1}{9}$
3. $T(p,q) = T(\frac{3}{2}, -\frac{1}{12})$ $\quad p = \frac{-b}{2a} = \frac{1}{\frac{2}{3}} = \frac{3}{2}, \quad q = \frac{-D}{4a} = \frac{-\frac{1}{9}}{\frac{4}{3}} = -\frac{1}{12}$
4. *ničli* :

$x_1 = \frac{-b+\sqrt{D}}{2a} = \frac{1+\frac{1}{3}}{\frac{2}{3}} = 2, \quad x_2 = \frac{-b-\sqrt{D}}{2a} = \frac{1-\frac{1}{3}}{\frac{2}{3}} = 1$



2.

$T(1,5)$
$a = -2$

$f(x) = a(x-p)^2 + q$
$= -2(x-1)^2 + 5 = -2(x^2 - 2x + 1) + 5 = 4x - 2x^2 + 3 = -2x^2 + 4x + 3$

REŠITEV:

$\mathbf{f(x) = -2x^2 + 4x + 3}$
$c = 3 \rightarrow$ Graf funkcije seka ordinatno os v točki $(0,3)$

3.

$T(1,-2)$
$A(3,2)$

$f(x) = a(x-p)^2 + q$
$2 = a(3-1)^2 - 2$
$2 = 4a - 2$
$4 = 4a$
$a = 1$

$f(x) = a(x-p)^2 + q$
$= (x-1)^2 - 2 = x^2 - 2x + 1 - 2 = x^2 - 2x - 1$

REŠITEV
$\mathbf{f(x) = x^2 - 2x - 1}$

4.

$T(2,-4)$
$A(3,1)$

$f(x) = a(x-p)^2 + q$
$1 = a(3-2)^2 - 4$
$1 = a - 4$
$5 = a$

$f(x) = a(x-p)^2 + q$
$= 5(x-2)^2 - 4 =$
$= 5(x^2 - 4x + 4) - 4 =$
$= 5x^2 - 20x + 16$

REŠITEV:
$\mathbf{f(x) = 5x^2 - 20x + 16}$

5.

ničli : $x_1 = 3,\ x_2 = -2$
A(4,-3)

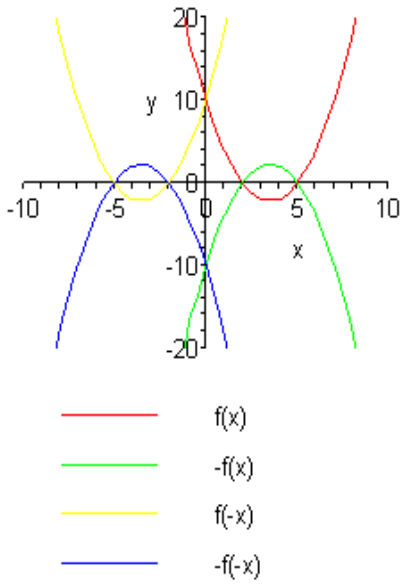
$f(x) = a(x-x_1)(x-x_2)$
$-3 = a(4-3)(4+2)$
$-3 = 6a$
$a = -\frac{1}{2}$

$f(x) = a(x-x_1)(x-x_2) =$
$= -\frac{1}{2}(x-3)(x+2) =$
$= -\frac{x^2}{2} + \frac{x}{2} + 3$

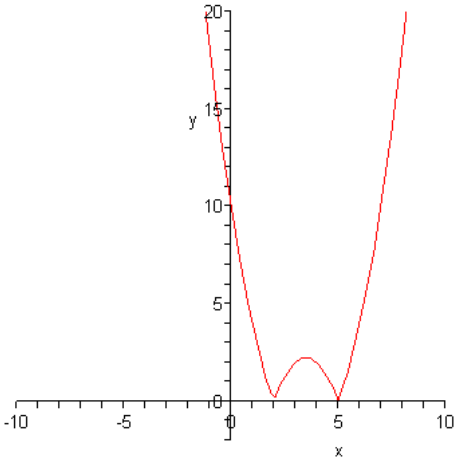
REŠITEV:
$\mathbf{f(x) = -\frac{x^2}{2} + \frac{x}{2} + 3}$

6.

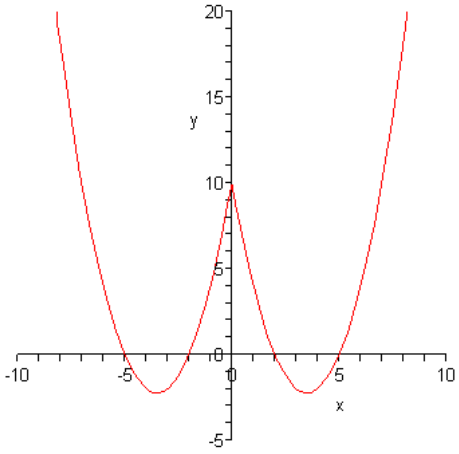
- $f(x) = x^2 - 7x + 10$
  $f(-x) = (-x)^2 - 7(-x) + 10 = x^2 + 7x + 10$
  $-f(x) = -(x^2 - 7x + 10) = -x^2 + 7x - 10$
  $-f(-x) = -((-x)^2 - 7(-x) + 10) = -x^2 - 7x - 10$



- $|f(x)| = |x^2 - 7x + 10|$
  $f(|x|) = |x|^2 - 7|x| + 10 = x^2 - 7|x| + 10$



$|f(x)|$



$f(|x|)$

7.

a)

$A(1,2) \quad \rightarrow 2 = a + b + c \rightarrow 2 = 2b + 4 \rightarrow 2 = 6a - 14 + 4 \rightarrow a = 2$
$B(-2,11) \quad \rightarrow 11 = 4a - 2b + c \rightarrow 11 = 3a - b + 4 \rightarrow b = 3a - 7 \rightarrow b = -1$
$C(-1,4) \quad \rightarrow 4 = a - b + c \quad \rightarrow c = 4 - a + b \rightarrow c = 1$

Rešitev:
$\mathbf{f(x) = 2x^2 - 1x + 1}$

b)

$A(3,-2) \quad \rightarrow -2 = 9a + 3b + c \rightarrow -2 = 8a + 2b \rightarrow -2 = 8b - 32 + 2b \rightarrow b = 3$
$B(-2,-12) \quad \rightarrow -12 = 4a - 2b + c \rightarrow -12 = 3a - 3b \rightarrow a = b - 4 \rightarrow a = -1$
$C(1,0) \quad \rightarrow 0 = a + b + c \rightarrow c = -a - b \rightarrow c = -2$

Rešitev:
$\mathbf{f(x) = -x^2 + 3x - 2}$

c)

$A(0,1) \quad \rightarrow c = 1$
$B(\frac{1}{2}, \frac{5}{3}) \quad \rightarrow \frac{5}{3} = \frac{1}{4}a + \frac{1}{2}b + 1 \rightarrow \frac{5}{3} = -\frac{1}{12} + \frac{1}{4}b + \frac{1}{2}b + 1 \rightarrow \frac{3}{4}b = \frac{20}{12} + \frac{1}{12} - \frac{12}{12} = \frac{3}{4} \rightarrow b = 1$
$C(-1, \frac{2}{3}) \quad \rightarrow \frac{2}{3} = a - b + 1 \rightarrow a = -\frac{1}{3} + b \rightarrow a = \frac{2}{3}$

Rešitev:
$\mathbf{f(x) = \frac{2}{3}x^2 + x + 1}$

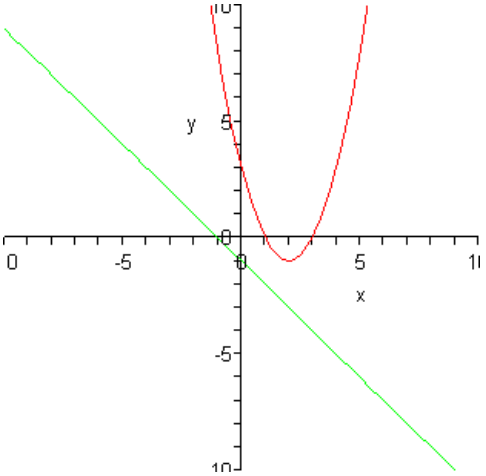
8.

a)

$f(x) = x^2 - 4x + 3 \rightarrow y = x^2 - 4x + 3 \rightarrow -x - 1 = x^2 - 4x + 3 \rightarrow -x^2 + 3x - 4 = 0$

$y = -x - 1$

se ne da realno razstaviti → ni presečišča



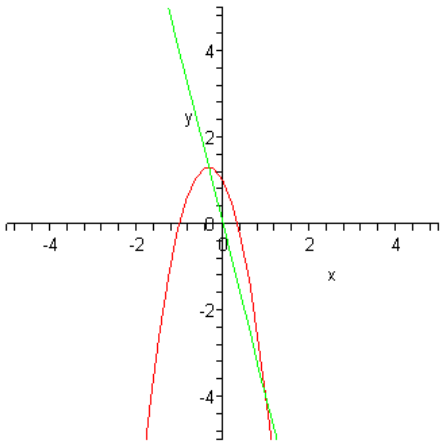
b)

$f(x) = -3x^2 - 2x + 1 \rightarrow -4x = -3x^2 - 2x + 1 \rightarrow -3x^2 + 2x + 1 = 0 \rightarrow (3x + 1)(-x + 1) = 0$

$y = -4x$

$x_1 = -\frac{1}{3}, y_1 = -4x_1 = \frac{4}{3} \rightarrow A(-\frac{1}{3}, \frac{4}{3})$

$x_2 = 1, y_2 = -4x_2 = -4 \rightarrow B(1, -4)$



Razdalja med presečiščema $\{A(-\frac{1}{3}, \frac{4}{3}), B(1, -4)\}$

$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 + \frac{1}{3})^2 + (-4 - \frac{4}{3})^2} = \frac{4}{3}\sqrt{17}$

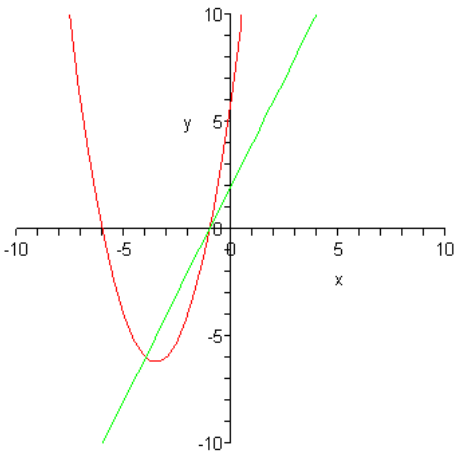
c)

$f(x) = x^2 + 7x + 6 \rightarrow 2x + 2 = x^2 + 7x + 6 \rightarrow x^2 + 5x + 4 = 0 \rightarrow (x+1)(x+4) = 0$

$y = 2x + 2$

$x_1 = -1, y_1 = 2x_1 + 2 = 0 \rightarrow A(-1, 0)$

$x_2 = -4, y_2 = 2x_2 + 2 = -6 \rightarrow B(-4, -6)$



Razdalja med presečiščema $\{A(-1,0), B(-4,-6)\}$

$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4+1)^2 + (-6)^2} = 3\sqrt{5}$

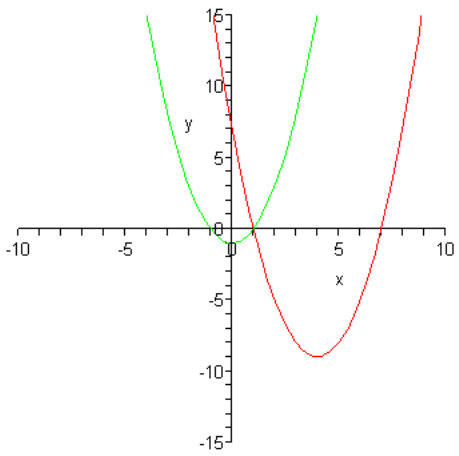
9.

a)

$f(x) = x^2 - 8x + 7 \rightarrow x^2 - 1 = x^2 - 8x + 7 \rightarrow x = 1$

$g(x) = x^2 - 1 \rightarrow\rightarrow\rightarrow\nearrow$

$y = x^2 - 1 = 0 \rightarrow A(1, 0)$



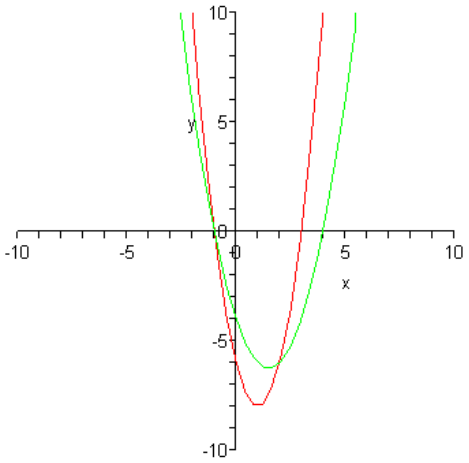
b)

$f(x) = 2x^2 - 4x - 6 \rightarrow x^2 - 3x - 4 = 2x^2 - 4x - 6 \rightarrow x^2 - x - 2 = 0 \rightarrow (x-2)(x+1) = 0$

$g(x) = x^2 - 3x - 4 \rightarrow \nearrow$

$x_1 = 2, y_1 = x_1^2 - 3x_1 - 4 = -6 \rightarrow A(2,-6)$

$x_2 = -1, y_2 = x_2^2 - 3x_2 - 4 = 0 \rightarrow B(-1,0)$



Ploščina trikotnika, ki ga določajo dve presečišči in koordinatno izhodišče:

$A(2,-6)$

$B(-1,0)$

$C(0,0)$

$$D = \pm\frac{1}{2}((x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)) = \pm\frac{1}{2}((-3)(6) - (-2)(6)) = \pm\frac{1}{2}(-6) = 3$$

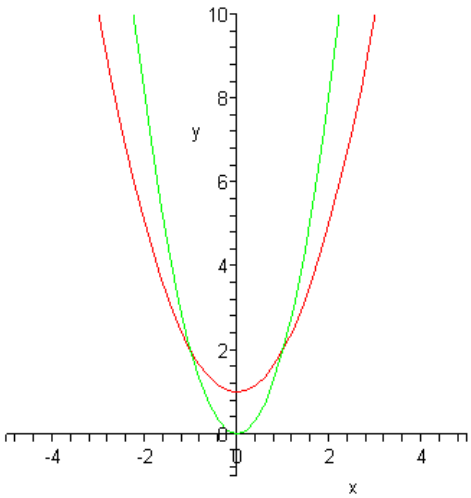
c)

$f(x) = x^2 + 1 \rightarrow 2x^2 = x^2 + 1 \rightarrow x^2 = 1$

$g(x) = 2x^2 \rightarrow\rightarrow\nearrow$

$x_1 = 1, y_1 = 2 \rightarrow A(1,2)$

$x_2 = -1, y_2 = 2 \rightarrow B(-1,2)$



Ploščina trikotnika, ki ga določajo dve presečišči in koordinatno izhodišče:

$A(1,2)$

$B(-1,2)$

$C(0,0)$

$$D = \pm\frac{1}{2}((x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)) = \pm\frac{1}{2}((-2)(-2) - (-1)(0)) = \pm\frac{1}{2}(4) = \mathbf{2}$$

10.

$f(x) = ax^2 - 2x - 5$ $\quad q = -2$

$T(p, -2)$ $\quad -2 = \frac{-D}{4a} = \frac{-(b^2-4ac)}{4a} = \frac{-(4+20a)}{4a} = \frac{-4-20a}{4a} = \frac{-1-5a}{a}$

$-2 = \frac{-1-5a}{a}$

$-2a = -1 - 5a$

$a = -\frac{1}{3}$

Za a=-$\frac{1}{3}$ je dosežena največja vrednost y=-2.

11.

$f(x) = ax^2 - (2a+1)x + 2(a+1)$ $\quad p = 2$

$T(2, q)$ $\quad 2 = \frac{-b}{2a} = \frac{2a+1}{2a}$

$4a = 2a + 1$

$a = \frac{1}{2}$

Za a=$\frac{1}{2}$ je dosežen minimum v x=2.

12.

$f(x) = x^2 + m(m+1)x + 100$ $\quad D = b^2 - 4ac$

Če D=0, se graf dotika x osi. $\quad 0 = m^4 + 2m^3 + m^2 - 400$

$0 = (m^2 + m)^2 - 400$

$0 = (m^2 + m + 20)(m^2 + m - 20)$

↙ ↘

Se ne da realno razstavit. $\quad (m+5)(m-4)$

$m_1 = -5$

$m_2 = 4$

Za vrednosti m=-5 in m=4 se graf dotika x osi.

13.

$f(x) = -x^2 - tx - 81$ $\quad D = b^2 - 4ac$

Če D=0 se graf dotika x osi. $\quad 0 = t^2 - 324$

$0 = (t-18)(t+18)$

$t_1 = 18$

$t_2 = -18$

Za vrednosti t=18 in t=-18 se graf dotika x osi.

14.

$y = ax^2 + 6x + c$ $\quad 8 = 4a + 12 + c \rightarrow 8 =\rightarrow -4 + 12 + c \rightarrow c = 0$

$A(2, 8)$ $\quad 0 = 36a + 36 + c \rightarrow 0 - 8 = 36a - 4a + 36 - 12 \rightarrow -32a = 32 \rightarrow a = -1$

ničla $x_1 = 6$

REŠITEV:

y = $-$x^2+6x

15.

$f(x) = (x-1)^2 + (x-2)^2 + (x-3)^2$
$= 3x^2 - 12x + 14$

$T(p,q)$ $\quad p = -\frac{b}{2a} = \frac{12}{6} = 2 \rightarrow$ Za vrednost x=2 zavzame funkcija najmanjšo vrednost.

$f(x) = (x-a_1)^2 + (x-a_2)^2 + \ldots + (x-a_n)^2 = x^2 + x^2 + \ldots + x^2 - 2(a_1 + a_2 + \ldots + a_n)x + a_1^2 + a_2^2 + \ldots + a_n^2$

$$p = -\frac{b}{2a} = \frac{2(a_1+a_2+\ldots+a_n)}{2(x^2+x^2+\ldots+x^2)} = \frac{(a_1+a_2+\ldots+a_n)}{(x^2+x^2+\ldots+x^2)} = \frac{(a_1+a_2+\ldots+a_n)}{n}$$

16.

$28 = x + (28 - x)$

1. *število* $= x = 14$
2. *število* $= 28 - x = 14$ $\quad f(x) = x^2 + (28-x)^2 = 2x^2 - 56x + 784$

$$p = \frac{-b}{2a} = \frac{56}{4} = 14 \rightarrow x = 14$$

17.

$a + b = -23 \rightarrow b = -23 - a$
$ab = 132 \rightarrow a(-23-a) = 132 \rightarrow -a^2 - 23a - 132 = 0 \rightarrow (-a-11)(a+12) = 0$
$a_1 = -11 \rightarrow b_1 = -12$
$a_2 = -12 \rightarrow b_2 = -11$

Števili, katerih vsota je -23, produkt pa 132 sta -11 in -12.

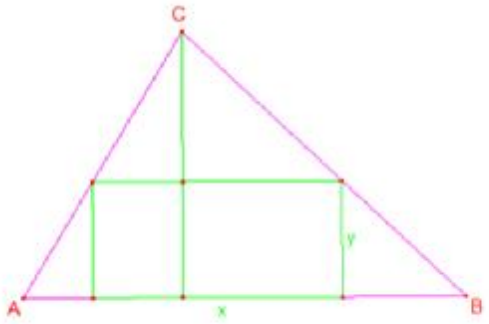
18.

$c^2 = x^2 + y^2 \quad x + y = 18 \rightarrow y = 18 - x$
$c^2 = x^2 + (18-x)^2 = 2x^2 - 36x + 324$

$p = -\frac{b}{2a} = \frac{36}{4} = 9 \rightarrow x = 9 \rightarrow y = 9$

Kateti sta enako dolgi.

19.



$x = x = 5$ $\qquad p_{pravokotnika} = x(6 - \frac{3}{5}x) = 5(6 - 3) = 15$

$y = 6 - \frac{3}{5}x = 3$

$f(x) = x(6 - \frac{3}{5}x) = -\frac{3}{5}x^2 + 6x \qquad p = \frac{-b}{2a} = \frac{-6}{-\frac{6}{5}} = 5 \rightarrow x = 5$

20.

x...obseg kvadrata→ $\frac{x}{4}\ldots$ stranica kvadrata $\rightarrow \frac{x^2}{16}\ldots$ ploščina kvadrata

56-x... obseg pravokotnika

v...višina pravokotnika

$$o_p = 2(3v + v)$$
$$56 - x = 8v$$
$$v = \frac{56-x}{8} \qquad \rightarrow \qquad 3v = 3(\frac{56-x}{8})$$

$p_p = 3v \times v$

$p_p = \frac{3(56-x)^2}{64}$

$p_k + p_p = \frac{x^2}{16} + \frac{3(56-x)^2}{64}$

$f(x) = \frac{x^2}{16} + \frac{3(3136-112x+x^2)}{64}$

$f(x) = \frac{4x^2+9408-336x+9408+3x^2}{64}$

$f(x) = \frac{7x^2}{64} - \frac{21}{4}x + 147 \qquad p = \frac{-b}{2a} = \frac{\frac{21}{4}}{\frac{14}{64}} = 24 \rightarrow x = 24$

Za kvadrat porabimo 24m žice, za pravokotnik pa 32.