

Vaje za 1. šolsko nalogo: 1. letnik- rešitve

1. Izpostavi skupni faktor:

- a) $18x + 27 = 9(2x + 3)$
- b) $4x^2 - 8x = 4x(x - 2)$
- c) $12x^2y - 8xy^2 = -4yx(-3x + 2y)$
- d) $36pq^3 - 12p^3q^2 + 60p^2q^4 - 24p^4q^3 = 12pq^2(3q - p^2 + 5pq^2 - 2p^3q)$
- e) $2x(x - 5)^2 - 3y(x - 5) = (x - 5)(2x^2 - 10x - 3y)$
- f) $4x(x - 3)^2 - 2y(x - 3) = 2(x - 3)(2x^2 - 6x - y)$
- g) $7x^3y^2 + 14x^2y - 42x^5y^3 + 21xy^4 = 7yx(x^2y + 2x - 6x^4y^2 + 3y^3)$
- h) $x^2(2x + 5) + 17(2x + 5) = (2x + 5)(x^2 + 17)$
- i) $6x^4 - 18x^3 + 12x^2 = 6x^2(x - 1)(x - 2)$
- j) $9x^4 - 18x^3 + 27x^2 = 9x^2(x^2 - 2x + 3)$

2. Razstavi po delih:

- a) $x^3 - 2x^2 + 5x - 10 = (x - 2)(x^2 + 5)$
- b) $x^3 - 3x^2 + 4x - 12 = (x - 3)(x^2 + 4)$
- c) $x^3 - x^2 + 2x - 2 = (x - 1)(x^2 + 2)$
- d) $x^3 + 6x^2 - 2x - 12 = (x + 6)(x^2 - 2)$
- e) $3x^3 - 2x^2 - 6x + 4 = (3x - 2)(x^2 - 2)$
- f) $a^2c + 5ac + 2a + 10 = (a + 5)(ac + 2)$
- g) $3y^2 + 4yz + 24y + 32z = (y + 8)(3y + 4z)$
- h) $4x^2y + 16xy - x - 4 = (x + 4)(4xy - 1)$
- i) $5x^2 + 15 - x^2y - 3y = -(x^2 + 3)(-5 + y)$

3. Zapiši kot produkt (razlika kvadratov):

- a) $x^2 - 100 = (x - 10)(x + 10)$
- b) $x^2 - 144 = (x - 12)(x + 12)$
- c) $36x^2 - 49 = (6x - 7)(6x + 7)$
- d) $64x^2 - 81 = (8x - 9)(8x + 9)$
- e) $9x^2 - 25y^2 = -(5y - 3x)(5y + 3x)$
- f) $9y^2 - (2x + 1)^2 = (3y + 1 + 2x)(3y - 1 - 2x)$
- g) $16y^2 - (3x - 1)^2 = (4y - 1 + 3x)(4y + 1 - 3x)$
- h) $x^4 - 81 = (x - 3)(x + 3)(x^2 + 9)$
- i) $x^4 - 16 = (x - 2)(x + 2)(x^2 + 4)$
- j) $81x^4 - 49y^6 = (-7y^3 + 9x^2)(7y^3 + 9x^2)$

4. Zapiši kot produkt (razlika, vsota kubov)

- a) $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$
- b) $x^3 - 64 = (x - 4)(x^2 + 4x + 16)$
- c) $y^3 + 27 = (y + 3)(y^2 - 3y + 9)$
- d) $y^3 + 125 = (y + 5)(y^2 - 5y + 25)$
- e) $64d^3 + 125 = (4d + 5)(16d^2 - 20d + 25)$
- f) $8d^3 + 27 = (2d + 3)(4d^2 - 6d + 9)$
- g) $125 - 8p^3d^3 = (5 - 2pd)(4p^2d^2 + 10pd + 25)$
- h) $64 - 27y^3z^3 = (4 - 3yz)(9y^2z^2 + 12yz + 16) =$

5. Razstavi tričlenik:

- a) $x^2 + 8x + 15 = (x + 5)(x + 3)$
- b) $x^2 - 8x + 15 = (x - 3)(x - 5)$
- c) $x^2 - 5x + 6 = (x - 2)(x - 3)$
- d) $x^2 + x - 30 = (x + 6)(x - 5)$
- e) $x^2 - 3x - 28 = (x + 4)(x - 7)$
- f) $25x^2 - 20x + 4 = (5x - 2)^2$
- g) $9x^2 - 12x + 4 = (3x - 2)^2$
- h) $2y^3 + 8y^2 + 8y = 2y(y + 2)^2$
- i) $3y^3 - 6y^2 + 3y = 3y(y - 1)^2$
- j) $4x^2 - 16x - 20 = 4(x + 1)(x - 5)$
- k) $3x^2 - 12x - 36 = 3(x + 2)(x - 6)$

6. Zapiši kot produkt:

- a) $x^2(x - 5) - 4(x - 5) = (x - 5)(x - 2)(x + 2)$
- b) $x^2(x + 3) - 81(x + 3) = (x + 9)(x + 3)(x - 9)$
- c) $y^3 - y = y(y - 1)(y + 1)$
- d) $c^3 - 16c = c(c - 4)(c + 4)$
- e) $x^2 + 8x + 16 - 49a^2 = (x + 4 + 7a)(x + 4 - 7a)$
- f) $16x - 2x^4 = -2x(x - 2)(x^2 + 2x + 4)$
- g) $7x^5y - 7xy^5 = 7yx(x - y)(y + x)(y^2 + x^2)$
- h) $y^3 - 2y^2 - 4y + 8 = (y + 2)(y - 2)^2$
- i) $12a^3b - 12ab^3 = -12ba(b - a)(b + a)$
- j) $r^3 - s^3 + r - s = (r - s)(r^2 + sr + 1 + s^2)$
- k) $7by^4 - 7b = 7b(y - 1)(y + 1)(y^2 + 1)$
- l) $x^3y^6 - x^3 = x^3(y - 1)(y + 1)(y^2 + y + 1)(y^2 - y + 1)$

7. Okrajšaj ulomke:

- a) $\frac{3x-9}{x^2-6x+9} = \frac{3}{x-3}$
- b) $\frac{4x-8}{x^2-4x+4} = \frac{4}{x-2}$
- c) $\frac{x^2-12x+36}{4x-24} = \frac{1}{4}x - \frac{3}{2}$
- d) $\frac{x^2-8x+16}{3x-12} = \frac{1}{3}x - \frac{4}{3}$
- e) $\frac{y^3-8}{y^2+2y-8} = \frac{y^2+2y+4}{y+4}$
- f) $\frac{x^3-9x}{x^3+5x^2+6x} = \frac{x-3}{x+2}$
- g) $\frac{x^3+x^2-20x}{x^3+2x^2-15x} = \frac{x-4}{x-3}$
- h) $\frac{y^2-4y-5}{y^2+5y+4} = \frac{y-5}{y+4}$

8. Izračunaj:

- a) $\frac{x^2-5x+6}{x^2-2x-3} \cdot \frac{x^2-1}{x^2-4} = \frac{x-1}{x+2}$
- b) $\frac{x^3-8}{x^2-4} \cdot \frac{x+2}{3x} = \frac{1}{3} \frac{x^2+2x+4}{x}$
- c) $\frac{x^2+5x+6}{x^2+x-6} \cdot \frac{x^2-9}{x^2-x-6} = \frac{x+3}{x-2}$
- d) $\frac{x^2+6x+9}{x^3+27} \cdot \frac{1}{x+3} = \frac{1}{x^2-3x+9}$
- e) $\frac{x^2-1}{(x-1)^2} \cdot \frac{x^3-1}{x+1} = x^2 + x + 1$
- f) $\frac{x^2+x-12}{x^2+x-30} \cdot \frac{x^2+5x+6}{x^2-2x-3} \cdot \frac{x^2+7x+6}{x+3} =$
- g) $\frac{x^2-4}{x^2+3x-10} \div \frac{x^2+5x+6}{x^2+8x+15} = 1$
- h) $\frac{y-1}{y^3-3y} \div \frac{1}{y^3-9y} = (y + 3)(y - 1)$

$$\begin{aligned} \text{i)} \quad & \frac{1-x^2}{6x-6} \div \frac{x^4-1}{6x^2+6} = -\frac{1}{x-1} \\ \text{j)} \quad & \frac{x^4-1}{2x} \pm \frac{x^3+x}{x^2} = \frac{1}{2x} (x^4 + 1 + 2x^2) \end{aligned}$$

9. Izračunaj:

$$\begin{aligned} \text{a)} \quad & \frac{4x-10}{x-2} - \frac{x-4}{x-2} = 3 \\ \text{b)} \quad & \frac{x^2-4x}{x^2-x-6} - \frac{x-6}{x^2-x-6} = \frac{x-2}{x+2} \\ \text{c)} \quad & \frac{3}{x+4} + \frac{6}{x+5} = 3 \frac{3x+13}{(x+4)(x+5)} \\ \text{d)} \quad & \frac{3}{x+1} - \frac{3}{x} = -\frac{3}{(x+1)x} \\ \text{e)} \quad & \frac{4}{x} - \frac{3}{x+3} = \frac{x+12}{x(x+3)} \\ \text{f)} \quad & \frac{2x}{x+2} + \frac{x+2}{x-2} = \frac{3x^2+4}{(x+2)(x-2)} \\ \text{g)} \quad & \frac{x+5}{x-5} + \frac{x-5}{x+5} = 2 \frac{x^2+25}{(x-5)(x+5)} \\ \text{h)} \quad & \frac{3x}{x^2+3x+10} - \frac{2x}{x^2+x-6} = x \frac{x^2-3x-38}{(x^2+3x+10)(x^2+x-6)} \\ \text{i)} \quad & \frac{x}{x^2-2x-24} - \frac{x}{x^2-7x+6} = -5 \frac{x}{(x^2-7x+6)(x+4)} \\ \text{j)} \quad & \frac{y+3}{y^2-y-2} - \frac{y-3}{y^2+2y+1} = 3 \frac{3y-1}{(y^2+2y+1)(y-2)} \\ \text{k)} \quad & \frac{y-3}{y-2} + \frac{7-4y}{2y^2-9y+10} - \frac{y+1}{2y-5} = \frac{y-12}{2y-5} \end{aligned}$$

10. Poenostavi in izračunaj:

$$\begin{aligned} \text{a)} \quad & \frac{\frac{x}{3}-1}{x-3} = \frac{1}{3} \\ \text{b)} \quad & \frac{1+\frac{1}{x}}{3-\frac{1}{x}} = \frac{x+1}{3x-1} \\ \text{c)} \quad & \frac{\frac{1}{x}+\frac{1}{y}}{x+y} = \frac{1}{xy} \\ \text{d)} \quad & \frac{1-\frac{1}{x}}{xy} = \frac{x-1}{x^2y} \\ \text{e)} \quad & \frac{\frac{y-2}{y^2-4} - \frac{y+2}{y^2-4}}{\frac{y-2}{y^2-4}} = -\frac{1}{7}y + 2 \\ \text{f)} \quad & \frac{\frac{y-2}{y^2-4} + 1}{\frac{3}{y^2-4} + 1} = 2 \frac{y+2}{y+1} \\ \text{g)} \quad & \frac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3} + \frac{1}{x-3}} = \frac{x-3}{x+2} \\ \text{h)} \quad & \frac{\frac{6}{x^2+2x-15} - \frac{1}{x-3}}{\frac{1}{x+5} + 1} = -\frac{x-1}{(6+x)(x-3)} \\ \text{i)} \quad & \frac{\frac{1}{x-y} - \frac{1}{x^2+xy+y^2}}{\frac{1}{x^3-125} - \frac{1}{x^2+5x+25}} = \frac{1}{x^2+xy+y^2-x+y} \\ \text{j)} \quad & \frac{\frac{1}{x^3-125}}{\frac{1}{x^2-25} - \frac{1}{x^2+5x+25}} = \frac{1}{5} \frac{x+5}{x+10} \end{aligned}$$

10. Določi največji skupni delitelj in najmanjši skupni večkratnik z razcepom na prafaktorje:

$$\begin{aligned} \text{a)} \quad & 245, 870; \quad 245 = 5 \cdot 7^2, 870 = 2 \times 3 \times 5 \times 29 \\ & \rightarrow D(245, 870) = 5, v(245, 870) = \frac{245 \cdot 870}{5} = 42\,630 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & 111, 297; \quad 111 = 3 \cdot 37, 297 = 3^3 \cdot 11 \\ & \rightarrow D(111, 297) = 3, v(111, 297) = \frac{111 \cdot 297}{3} = 10\,989 \end{aligned}$$

Delitelj preveri z Evklidovim algoritmom.

11. Določi pare $\{m, n\}$ naravnih števil, za katere je $D(m, n) = 5$ in $v(m, n) = 105$.

$$R = \{(m, n); (5, 105), (15, 105), (15, 35), (35, 15), (35, 105), (105, 5), (105, 15), (105, 35)\}$$

12. Zobnik z 78 zobmi poganja drugi zobnik z 48 zobmi. Po koliko obratih najšega zobnika bo izbrani zob prvič spet v isti vrzeli kot na začetku?

Rešitev je najmanjši skupni večkratnik $v(78, 48) = 2^4 \cdot 3 \cdot 13 = 624$

13. Izračunaj x :

a) $1210_{(x)} = 48$;

b) $150_{(x)} = 66$,

c) $321_{(x)} = 200_{(9)}$

d) $12_{(3)} + 221_{(5)} = x_{(4)}$

e) $101101_{(2)} + 11011_{(2)} = x_{(2)}$

f) $110001_{(2)} \cdot 10101_{(2)} = x_{(2)}$

g) $100101111_{(2)} = x_{(n)}$, kjer je $n \in \{4, 8, 10\}$

14. Naj bodo množice $A = \{0, 1, 2, 4, 5, 7\}$, $B = \{0, 3, 4, 5, 8, 9\}$, $C = \{0, 3, 6, 7\}$.

Določi:

a) $(A \cup B) \cap C = \{0, 3, 7\}$ b) $(A \cup B) \cup C = \mathbb{N}_9 \cup \{0\}$

c) $(A \cup B) \cap (C \cup B) = \{0, 3, 4, 5, 7, 8, 9\}$

d) $(A \setminus B) \cup C = \{0, 1, 2, 3, 6, 7\}$ e) $((A \cup B) \cup C) \setminus (A \cap C) = \{1, 2, 3, 4, 5, 6, 8, 9\}$

f) Napiši potenčno množico $C \setminus \{7\}$.

$P(\{0, 3, 6\}) = \{\emptyset, \{0\}, \{3\}, \{6\}, \{0, 3\}, \{0, 6\}, \{3, 6\}, \{0, 3, 6\}\}$

g) Zapiši kartezični produkt $(A \setminus \{4, 5, 7\}) \times C$

$(A \setminus \{4, 5, 7\}) \times C = \{0, 1, 2\} \times \{0, 3, 6, 7\} =$

$= \{(0, 0), (0, 3), (0, 6), (0, 7), (1, 0), (1, 3), (1, 6), (1, 7), (2, 0), (2, 3), (2, 6), (2, 7)\}$

15. V razredu je 40 učencev. 4 učenci poslušajo jazz, klasiko in rock. 7 učencev je takih, ki glasbe sploh ne poslušajo. Dva poslušata le klasiko, trije poslušajo le jazz, enajst pa le rock. Za samo jazz in klasiko se zanima le 1, za samo klasiko in rock pa 5 učencev.

Koliko učencev poslušata jazz in rock in ne poslušata klasike? $R=7$

Koliko učencev poslušata klasiko ali jazz? $R=22$

16. Določi $A \cup B$ in $A \cap B$, če sta množici $A = \{5n; n \in \mathbb{N}\}$ in $B = \{15n; n \in \mathbb{N}\}$.

$A \cup B = A$

$A \cap B = B$