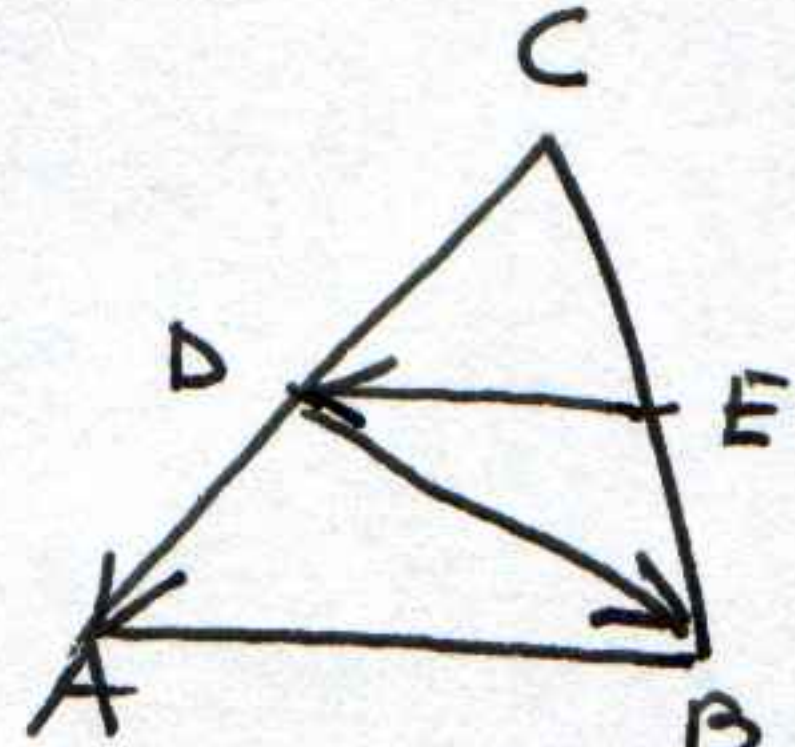


ŘEŠITĚ:

①



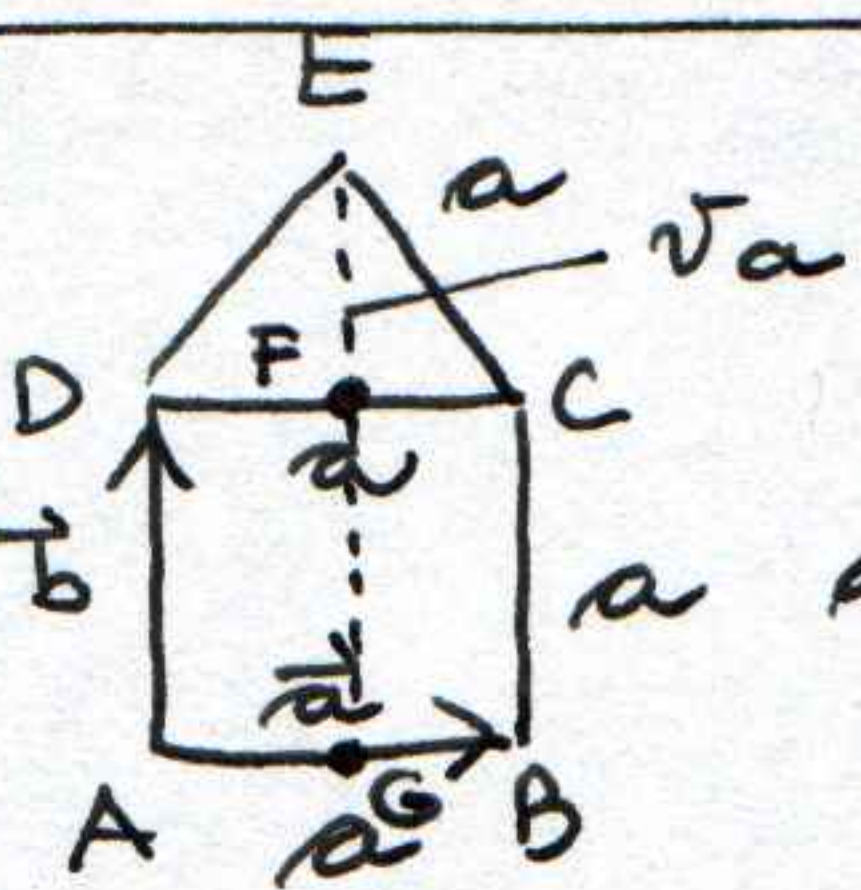
$$\vec{AB} = \vec{c} \quad \vec{BC} = \vec{a} \quad \vec{AC} = \vec{b}$$

$$\vec{ED} = \vec{EC} + \vec{CD} = \frac{1}{2}\vec{BC} + \frac{1}{2}\vec{CA} = \frac{1}{2}\vec{a} - \frac{1}{2}\vec{b}$$

$$\vec{DA} = -\frac{1}{2}\vec{c}$$

$$\vec{DB} = -\frac{1}{2}\vec{c} + \vec{a}$$

②



$$\vec{BC} = \vec{AD} = \vec{GF} \quad \vec{FE} = m\vec{GF} = m\vec{BC} \Rightarrow \vec{FE} = \frac{\sqrt{3}}{2}\vec{AD} = \frac{\sqrt{3}}{2}\vec{b}$$

$$\vec{DE} = \frac{1}{2}\vec{a} + \vec{FE} = \frac{1}{2}\vec{a} + \frac{\sqrt{3}}{2}\vec{b}$$

$$\vec{CE} = -\frac{1}{2}\vec{a} + \vec{FE} = -\frac{1}{2}\vec{a} + \frac{\sqrt{3}}{2}\vec{b}$$

enokotrh. Δ !

③

$$2m\vec{a} - 8m\vec{b} + 3\vec{a} = 10m\vec{b} - 7\vec{b} - m\vec{a}$$

$$(2m+3+m)\vec{a} + (-8m-10m+7)\vec{b} = 0$$

$$\begin{cases} 2m+m+3=0 \\ -8m-10m+7=0 \end{cases} \Rightarrow \begin{cases} 2m+m = -3 \\ -10m-8m = -7 \end{cases} \quad \cdot 5 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} +$$

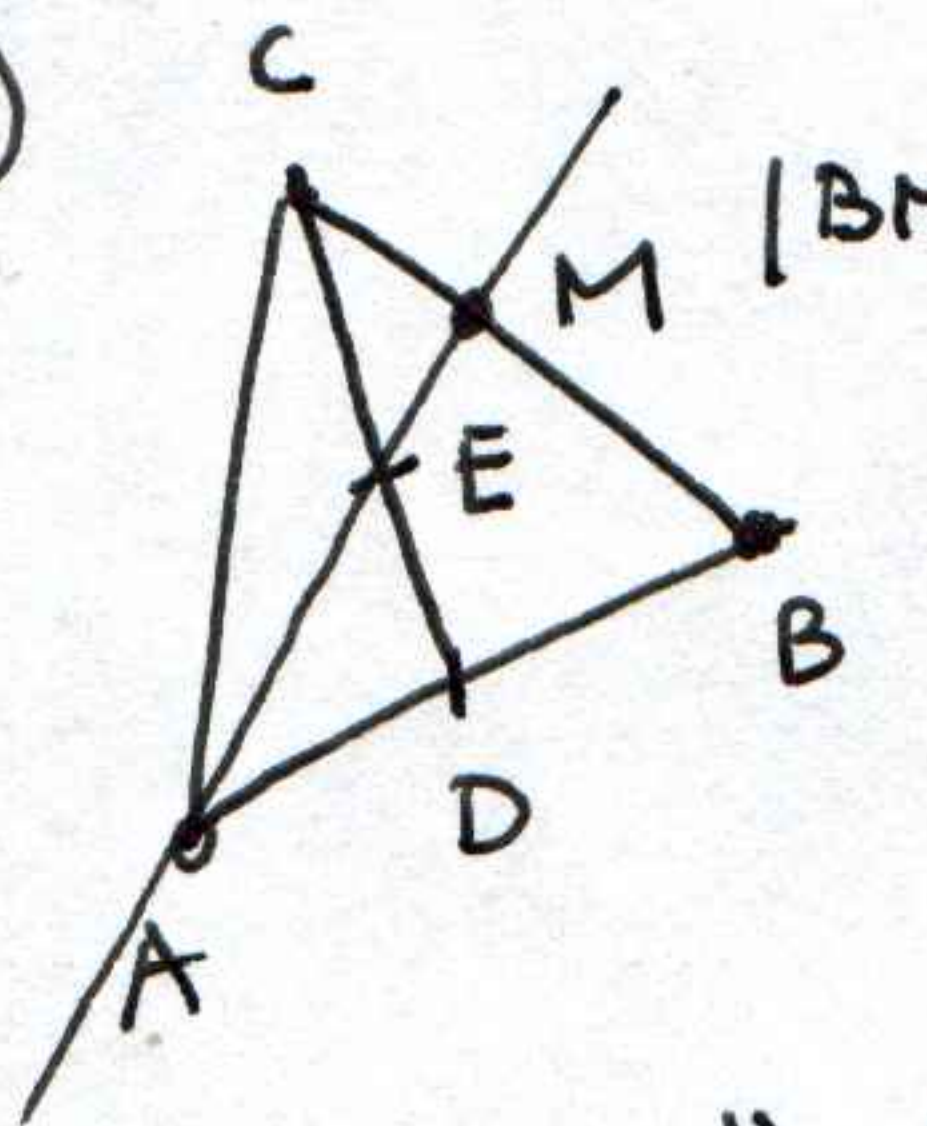
$$m = \frac{-3-m}{2} = \frac{-3-1}{6} = m$$

$$5m-8m = -15-7$$

$$-3m = -22$$

$$m = \frac{22}{3}$$

④



$$|BM|:|MC| = ? = [2:1]$$

$$\vec{AB} = \vec{c} \quad \vec{BC} = \vec{a} \quad \vec{AC} = \vec{b}$$

$$\vec{BM} = m\vec{BC} = m(\vec{b} - \vec{c})$$

$$\vec{ME} = -\vec{c} + t\vec{AE} = -\vec{c} + t(\vec{AD} + \vec{DE}) = -\vec{c} + t(\frac{1}{2}\vec{c} + \frac{1}{2}(\vec{b} - \frac{1}{2}\vec{c}))$$

$$m\vec{b} - m\vec{c} = -\vec{c} + \frac{t}{2}\vec{c} + \frac{t}{2}\vec{b} - \frac{t}{4}\vec{c}$$

$$(m - \frac{t}{4})\vec{b} + (-m + 1 - \frac{t}{2} + \frac{t}{4})\vec{c} = 0$$

$$m = \frac{t}{2} \quad 1 = m + \frac{t}{4} \Rightarrow t = \frac{4}{3}$$

$$\vec{BM} = \frac{2}{3}\vec{BC}$$

$$m = \frac{2}{3}$$

ali:

$$\vec{CA} = \vec{b} \quad \vec{CB} = \vec{a}$$

$$\vec{CM} = m\vec{CB} = m\vec{a}$$

$$\vec{CE} = \vec{CA} + t\vec{AE} = \vec{b} + t(-\vec{b} + \frac{1}{2}(\frac{1}{2}\vec{a} + \vec{b}))$$

$$m\vec{a} = \vec{b} - t\vec{b} + \frac{t}{4}\vec{a} + \frac{t}{4}\vec{b}$$

$$(m - \frac{t}{4})\vec{a} + (t - \frac{t}{4} - 1)\vec{b} = 0$$

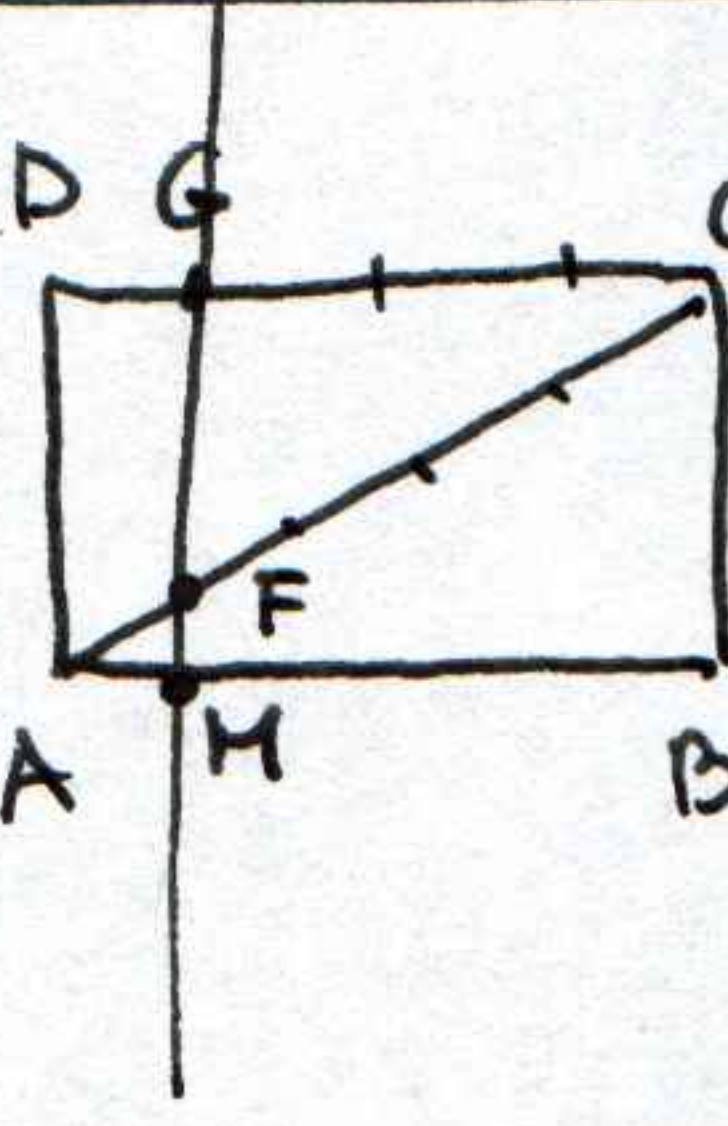
$$m = \frac{t}{4} \quad \frac{3}{4}t = 1 \Rightarrow t = \frac{4}{3}$$

$$m = \frac{1}{3}$$

$$\vec{CM} = \frac{1}{3}\vec{CB}$$

$$|CM|:|MB| = 1:2$$

⑤



$$|AM|:|MB| = ?$$

$$\vec{AB} = \vec{a} \quad \vec{AD} = \vec{b}$$

$$\vec{AM} = m\vec{AB} = m\vec{a}$$

$$\vec{AF} = \vec{AF} + \vec{FM} = \frac{1}{5}\vec{AC} + t\vec{GF} = \frac{1}{5}(\vec{a} + \vec{b}) + t(-\frac{1}{4}\vec{a} - \vec{b} + \frac{1}{5}(\vec{a} + \vec{b}))$$

$$\vec{AF} = \frac{3}{16}\vec{AB} \Rightarrow |AM|:|MB| = 3:13$$

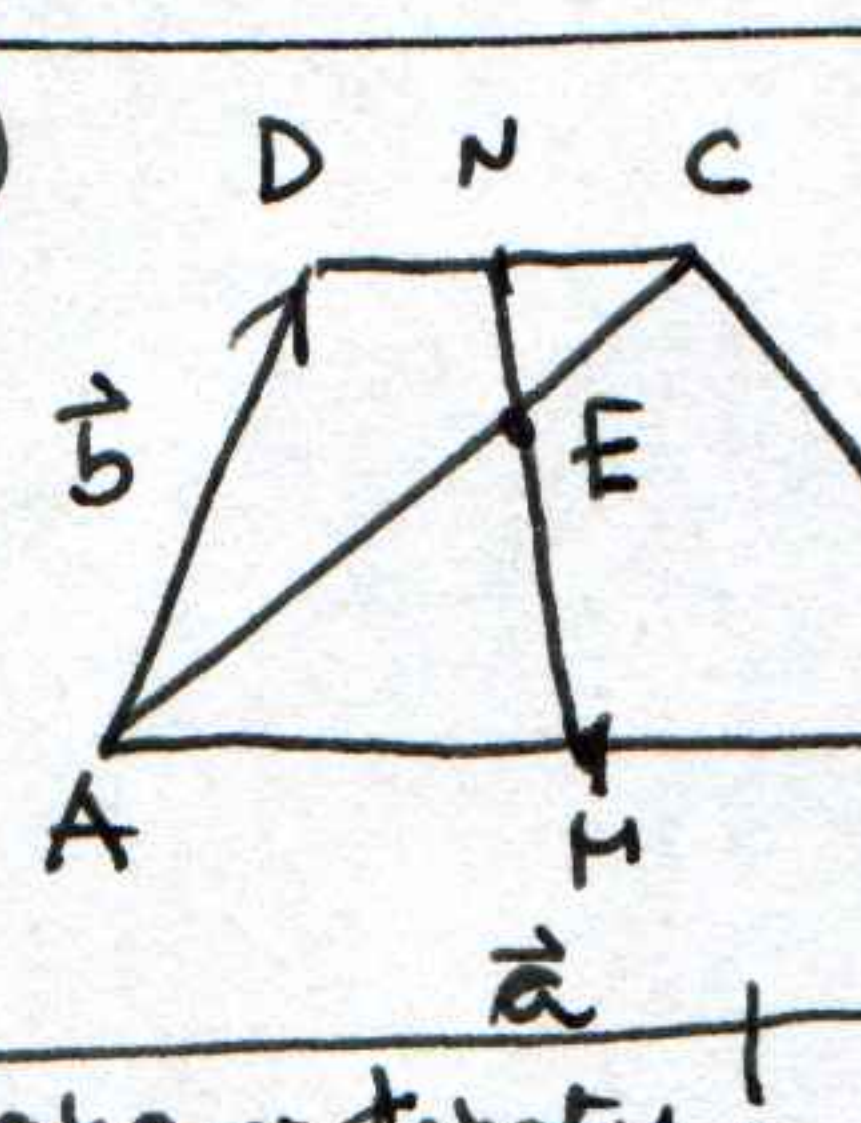
$$m\vec{a} = \frac{1}{5}\vec{a} + \frac{1}{5}\vec{b} - \frac{t}{4}\vec{a} - t\vec{b} + \frac{t}{5}\vec{a} + \frac{t}{5}\vec{b}$$

$$(m - \frac{1}{5} + \frac{t}{4} - \frac{t}{5})\vec{a} + (-\frac{1}{5} + t - \frac{t}{5})\vec{b} = 0$$

$$\frac{1}{5} = m + \frac{t}{20} \quad \frac{1}{5} = \frac{4}{5}t \Rightarrow t = \frac{1}{4}$$

$$m = \frac{1}{5} - \frac{t}{20} = \frac{1}{5} - \frac{1}{80} = \frac{15}{80} = \frac{3}{16}$$

⑥



$$\vec{DC} = \frac{1}{2}\vec{AB} = \frac{1}{2}\vec{a}$$

$$|AE|:|EC| = ? [2:1]$$

$$\vec{AE} = m\vec{AC} = m(\vec{c} + \frac{1}{2}\vec{a})$$

$$\vec{AE} = \vec{AH} + \vec{HE} = \frac{1}{2}\vec{a} + t\vec{HN} = \frac{1}{2}\vec{a} + t(-\frac{1}{2}\vec{a} + \vec{c} + \frac{1}{4}\vec{a})$$

$$\vec{AE} = \frac{2}{3}\vec{AC} \Rightarrow |AE|:|EC| = 2:1$$

$$m\vec{b} + \frac{1}{2}m\vec{a} = \frac{1}{2}\vec{a} - \frac{t}{2}\vec{a} + t\vec{b} + \frac{t}{4}\vec{a}$$

$$(\frac{m}{2} - \frac{1}{2} + \frac{t}{2} - \frac{t}{4})\vec{a} + (m - t)\vec{b} = 0$$

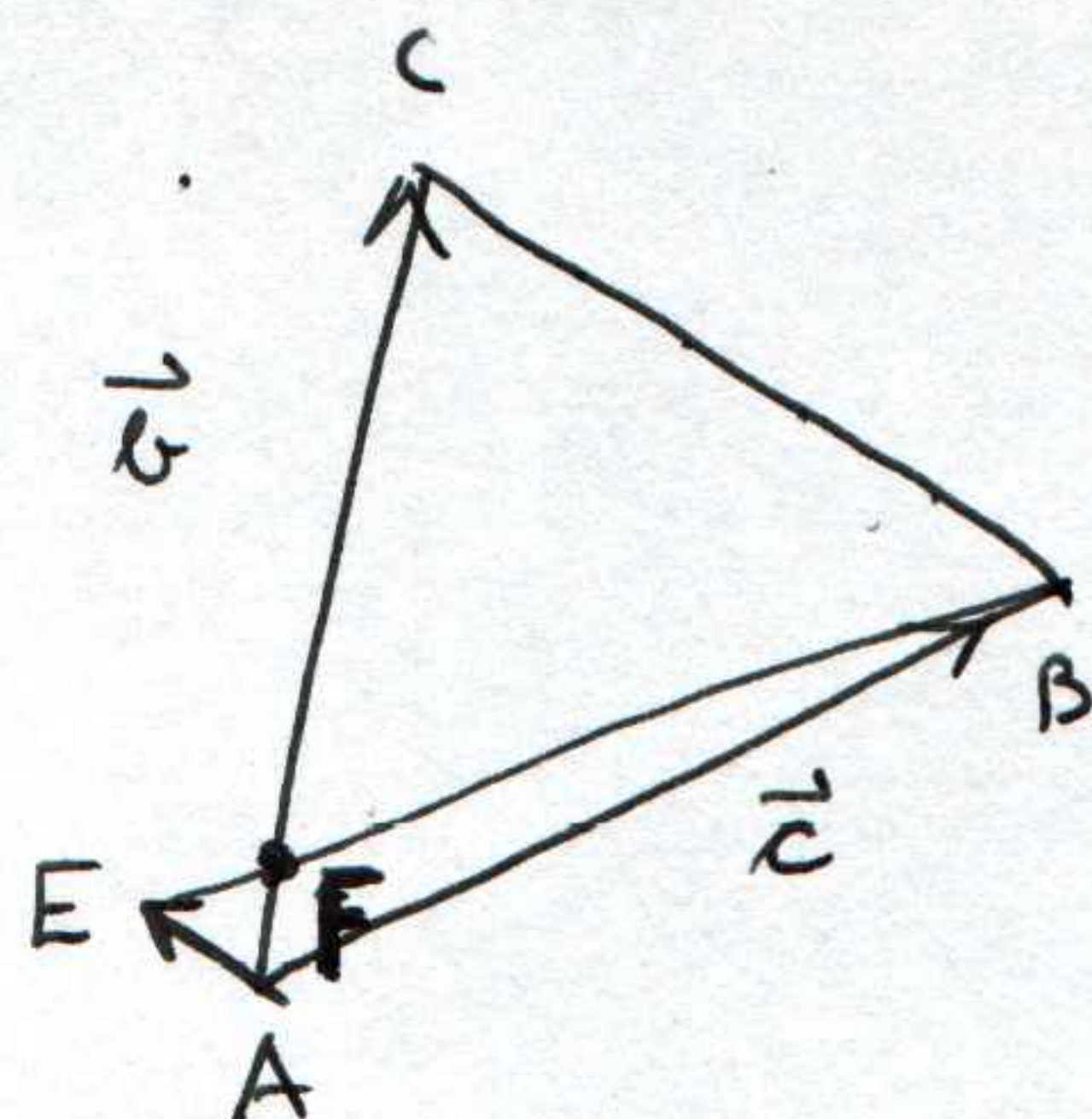
$$\frac{m}{2} + \frac{t}{4} = \frac{1}{2} \quad ; \quad m = t$$

$$\frac{m}{2} + \frac{m}{4} = \frac{1}{2}$$

$$\frac{3m}{4} = \frac{1}{2} \quad m = \frac{4}{3} \cdot \frac{1}{2} = \frac{2}{3} = t$$

kytaka v terstu!
délžime osnovnice
AB.... (popravi!)

7



$$\vec{AE} = \frac{1}{5} \vec{BC} = \frac{1}{5} (\vec{c} - \vec{c})$$

$$|AF|:|FC| = ? \quad [1:5]$$

$$\Rightarrow m\vec{b} = \frac{1}{5}\vec{b} - \frac{1}{5}\vec{c} - \frac{t}{5}\vec{b} + \frac{t}{5}\vec{c} + t\vec{c}$$

$$(m - \frac{1}{5} + \frac{t}{5})\vec{b} + (\frac{1}{5} - \frac{t}{5} - t)\vec{c} = 0$$

$$m + \frac{t}{5} = \frac{1}{5}$$

$$\frac{1}{5} = \frac{6t}{5} \Rightarrow t = \frac{1}{6}$$

$$\vec{AF} = m\vec{AC} = m\vec{c}$$

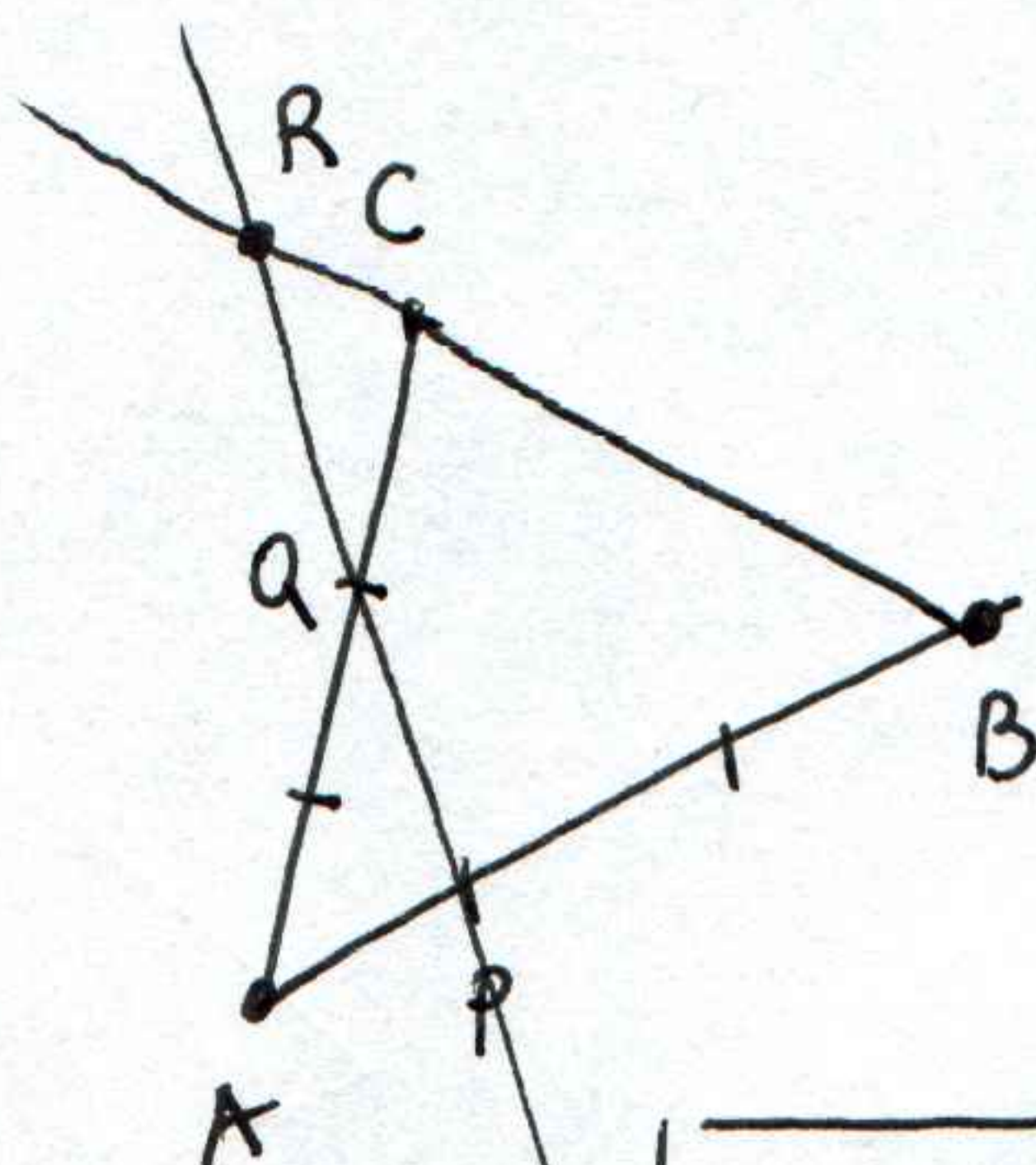
$$\vec{AF} = \vec{AE} + \vec{EF} = \frac{1}{5}\vec{BC} + t\vec{EB} = \frac{1}{5}(\vec{b} - \vec{c}) + t(-\frac{1}{5}\vec{b} + \vec{c})$$

$$= \frac{1}{5}\vec{b} - \frac{1}{5}\vec{c} + t(-\frac{1}{5}\vec{b} + \vec{c})$$

$$m = \frac{1}{5} - \frac{t}{5}$$

$$m = \frac{1}{6}$$

8



$$|BC|:|CR| = ? \quad [4:5]$$

$$\vec{BR} = m\vec{BC} = m(\vec{b} - \vec{c})$$

$$\vec{BR} = -\frac{2}{3}\vec{c} + t\vec{PQ} = -\frac{2}{3}\vec{c} + t(-\frac{2}{3}\vec{b} + \frac{1}{3}\vec{a})$$

$$m\vec{b} - m\vec{c} = -\frac{2}{3}\vec{c} - \frac{2}{3}t\vec{b} + \frac{t}{3}\vec{c}$$

$$(m + \frac{2}{3}t)\vec{b} + (-m + \frac{2}{3} - \frac{t}{3})\vec{c} = 0$$

$$m = \frac{2}{3}t$$

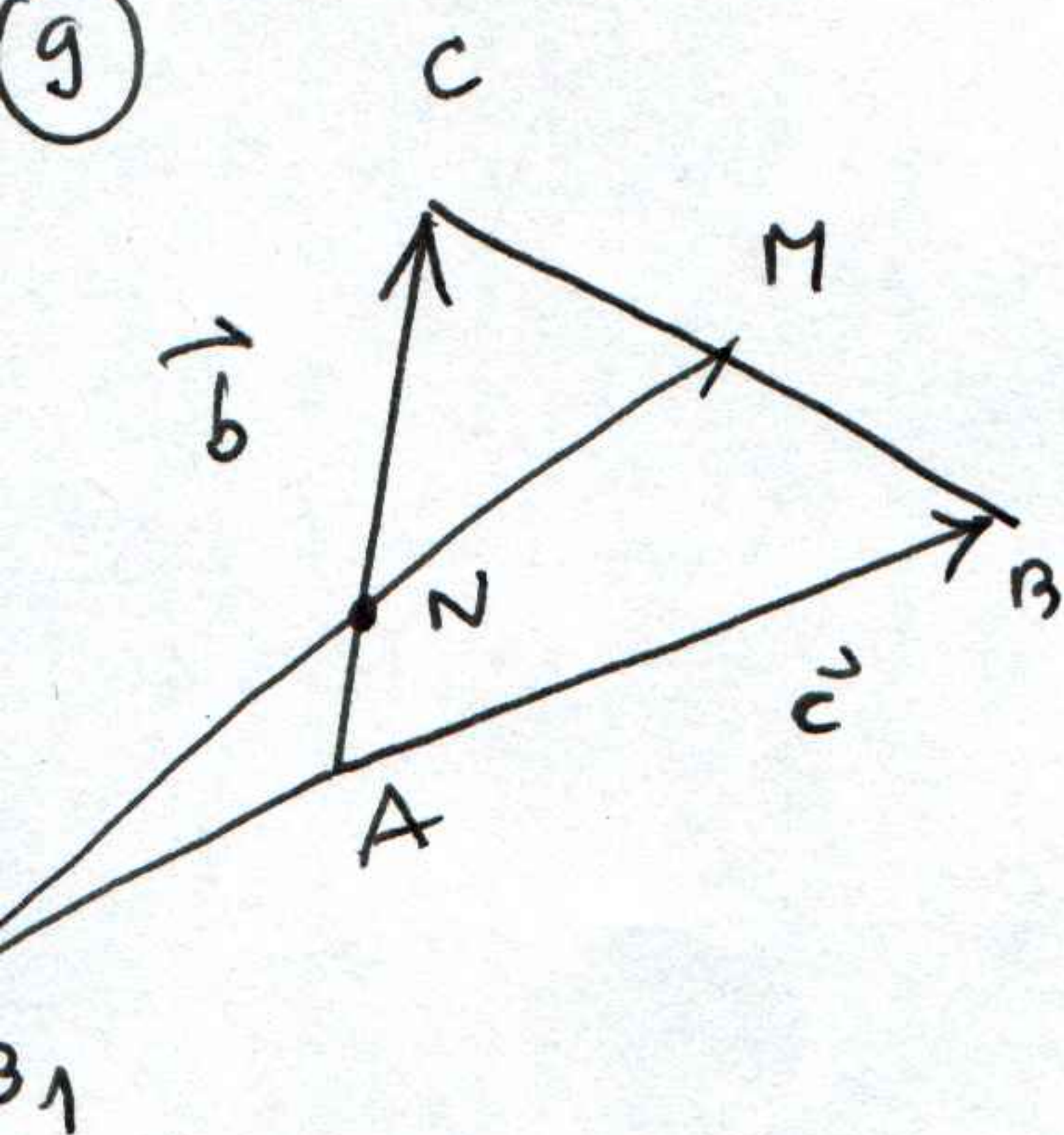
$$m + \frac{t}{3} = \frac{2}{3}$$

$$t = \frac{2}{3} \quad m = \frac{4}{9}$$

NAPAKA U TEKSTU:

$$\text{BIDJI MORAO: } |PB| = 2|AP|$$

9



a) M je razpolovnica BC

$$|AN|:|NC| = ?$$

$$\vec{AN} = m\vec{AC} = m\vec{c}$$

$$\vec{AN} = -\vec{c} + \vec{B_1N}$$

$$= -\vec{c} + t\vec{B_1M}$$

$$= -\vec{c} + t(2\vec{c} + \frac{1}{2}(\vec{b} - \vec{c})) \Rightarrow$$

$$\vec{AN} = \frac{1}{3}\vec{AC} \Rightarrow |AN|:|NC| = 1:2$$

$$m\vec{b} = -\vec{c} + 2t\vec{c} + \frac{t}{2}\vec{b} - \frac{t}{2}\vec{c}$$

$$(m - \frac{t}{2})\vec{b} + (1 - 2t + \frac{t}{2})\vec{c} = 0$$

$$m = \frac{t}{2}$$

$$1 = 2t - \frac{t}{2}$$

$$1 = \frac{3t}{2}$$

$$t = \frac{2}{3} \quad m = \frac{1}{3}$$

b) $|BM|:|BC| = k:1 \Rightarrow k < 0$

$$\vec{BM} = k\vec{BC}$$

$$\vec{AN} = m\vec{AC} = m\vec{c}$$

$$\vec{AN} = -\vec{c} + t(2\vec{c} + k\vec{BC}) =$$

$$= -\vec{c} + t(2\vec{c} + k(\vec{b} - \vec{c}))$$

$\Rightarrow$

$$m\vec{b} = -\vec{c} + 2t\vec{c} + t \cdot k\vec{b} - t \cdot k\vec{c}$$

$$(m - t \cdot k)\vec{b} + (1 - 2t + t \cdot k)\vec{c} = 0$$

$$m = t \cdot k$$

$$1 = 2t - t \cdot k$$

$$1 = t(2 - k)$$

$$t = \frac{1}{2 - k}$$

$$m = \frac{k}{2 - k}$$

a) je poseben primer b). Če v b) vzamemo $k = \frac{1}{2} \Rightarrow$ a)

$$t = \frac{1}{2 - \frac{1}{2}} = \frac{2}{3} \checkmark$$

$$m = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} \checkmark$$