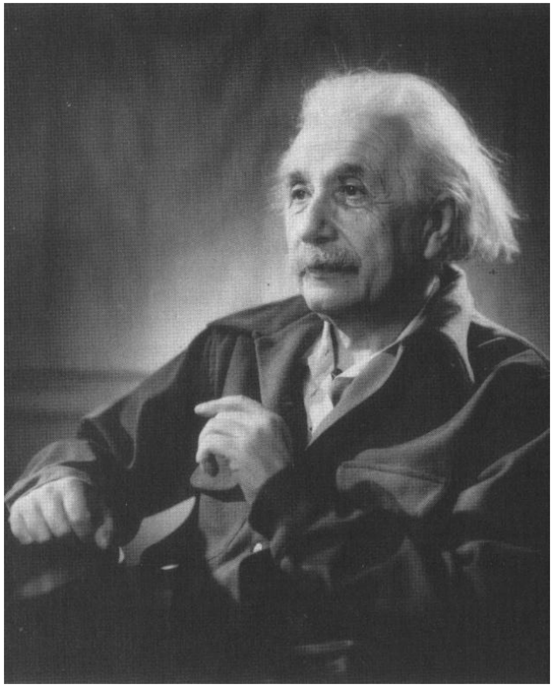
Marko Uršič, Filozofija narave, FF

Osnove Einsteinove teorije relativnosti

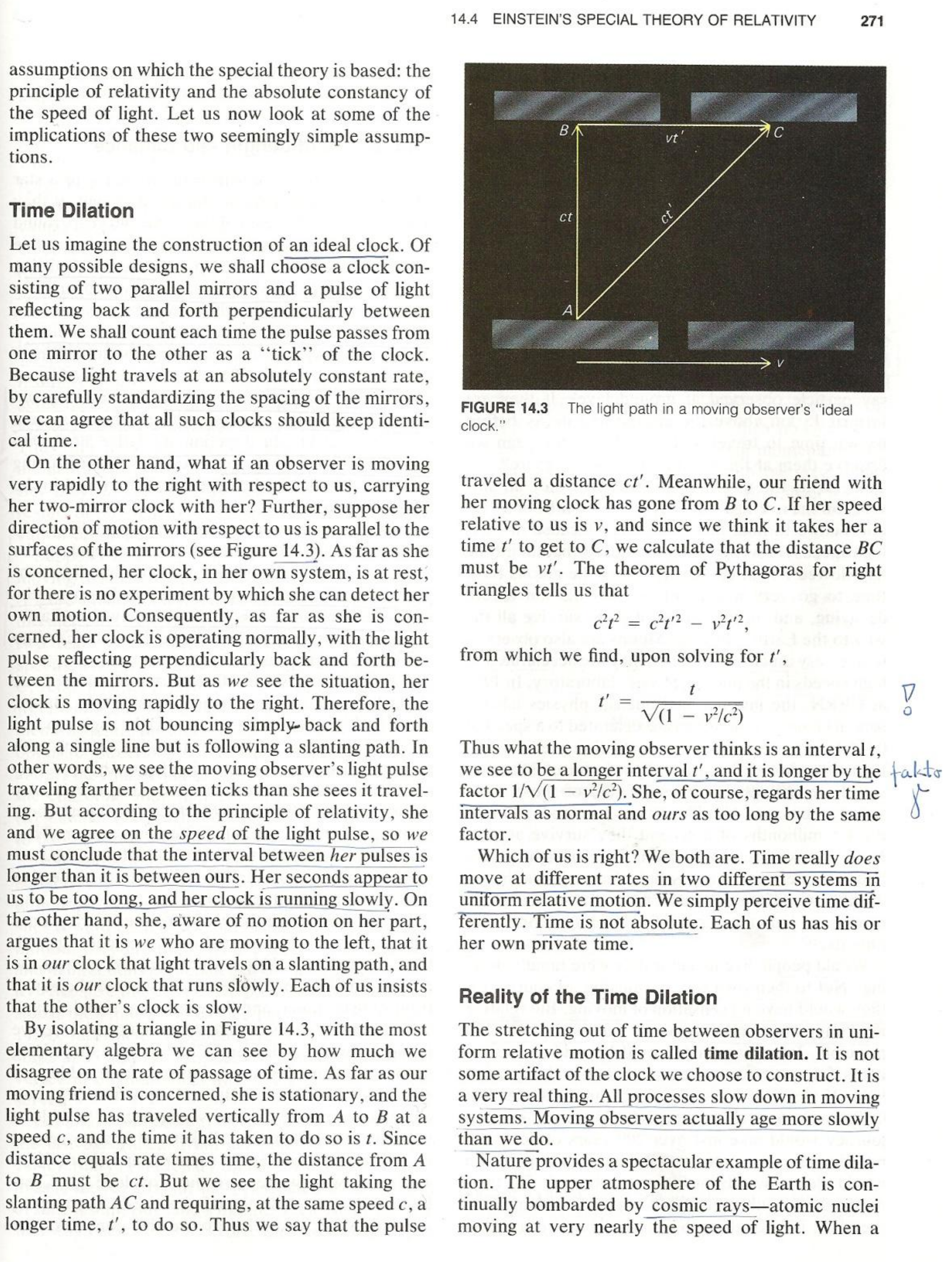
predavanja za študente filozofije

(gradivo z zaslona)



Albert Einstein
(1879–1955)

1905: posebna (ali specialna) teorija relativnosti (SRT)
1916: splošna (ali generalna) teorija relativnosti (GRT)



14.4 EINSTEIN'S SPECIAL THEORY OF RELATIVITY 271

assumptions on which the special theory is based: the principle of relativity and the absolute constancy of the speed of light. Let us now look at some of the implications of these two seemingly simple assumptions.

Time Dilation

Let us imagine the construction of an ideal clock. Of many possible designs, we shall choose a clock consisting of two parallel mirrors and a pulse of light reflecting back and forth perpendicularly between them. We shall count each time the pulse passes from one mirror to the other as a "tick" of the clock. Because light travels at an absolutely constant rate, by carefully standardizing the spacing of the mirrors, we can agree that all such clocks should keep identical time.

On the other hand, what if an observer is moving very rapidly to the right with respect to us, carrying her two-mirror clock with her? Further, suppose her direction of motion with respect to us is parallel to the surfaces of the mirrors (see Figure 14.3). As far as she is concerned, her clock, in her own system, is at rest, for there is no experiment by which she can detect her own motion. Consequently, as far as she is concerned, her clock is operating normally, with the light pulse reflecting perpendicularly back and forth between the mirrors. But as *we* see the situation, her clock is moving rapidly to the right. Therefore, the light pulse is not bouncing simply back and forth along a single line but is following a slanting path. In other words, we see the moving observer's light pulse traveling farther between ticks than she sees it traveling. But according to the principle of relativity, she and we agree on the *speed* of the light pulse, so *we* must conclude that the interval between *her* pulses is longer than it is between ours. Her seconds appear to us to be too long, and her clock is running slowly. On the other hand, she, aware of no motion on her part, argues that it is *we* who are moving to the left, that it is in *our* clock that light travels on a slanting path, and that it is *our* clock that runs slowly. Each of us insists that the other's clock is slow.

By isolating a triangle in Figure 14.3, with the most elementary algebra we can see by how much we disagree on the rate of passage of time. As far as our moving friend is concerned, she is stationary, and the light pulse has traveled vertically from A to B at a speed c, and the time it has taken to do so is t. Since distance equals rate times time, the distance from A to B must be ct. But we see the light taking the slanting path AC and requiring, at the same speed c, a longer time, t', to do so. Thus we say that the pulse traveled a distance ct'. Meanwhile, our friend with her moving clock has gone from B to C. If her speed relative to us is v, and since we think it takes her a time t' to get to C, we calculate that the distance BC must be vt'. The theorem of Pythagoras for right triangles tells us that

$$c^2t^2 = c^2t'^2 - v^2t'^2,$$

from which we find, upon solving for t',

$$t' = \frac{t}{\sqrt{(1 - v^2/c^2)}}$$!

Thus what the moving observer thinks is an interval t, we see to be a longer interval t', and it is longer by the factor $1/\sqrt{(1 - v^2/c^2)}$. She, of course, regards her time intervals as normal and *ours* as too long by the same factor.

fakto
γ

Which of us is right? We both are. Time really *does* move at different rates in two different systems in uniform relative motion. We simply perceive time differently. Time is not absolute. Each of us has his or her own private time.

FIGURE 14.3 The light path in a moving observer's "ideal clock."

Reality of the Time Dilation

The stretching out of time between observers in uniform relative motion is called **time dilation.** It is not some artifact of the clock we choose to construct. It is a very real thing. All processes slow down in moving systems. Moving observers actually age more slowly than we do.

Nature provides a spectacular example of time dilation. The upper atmosphere of the Earth is continually bombarded by cosmic rays—atomic nuclei moving at very nearly the speed of light. When a

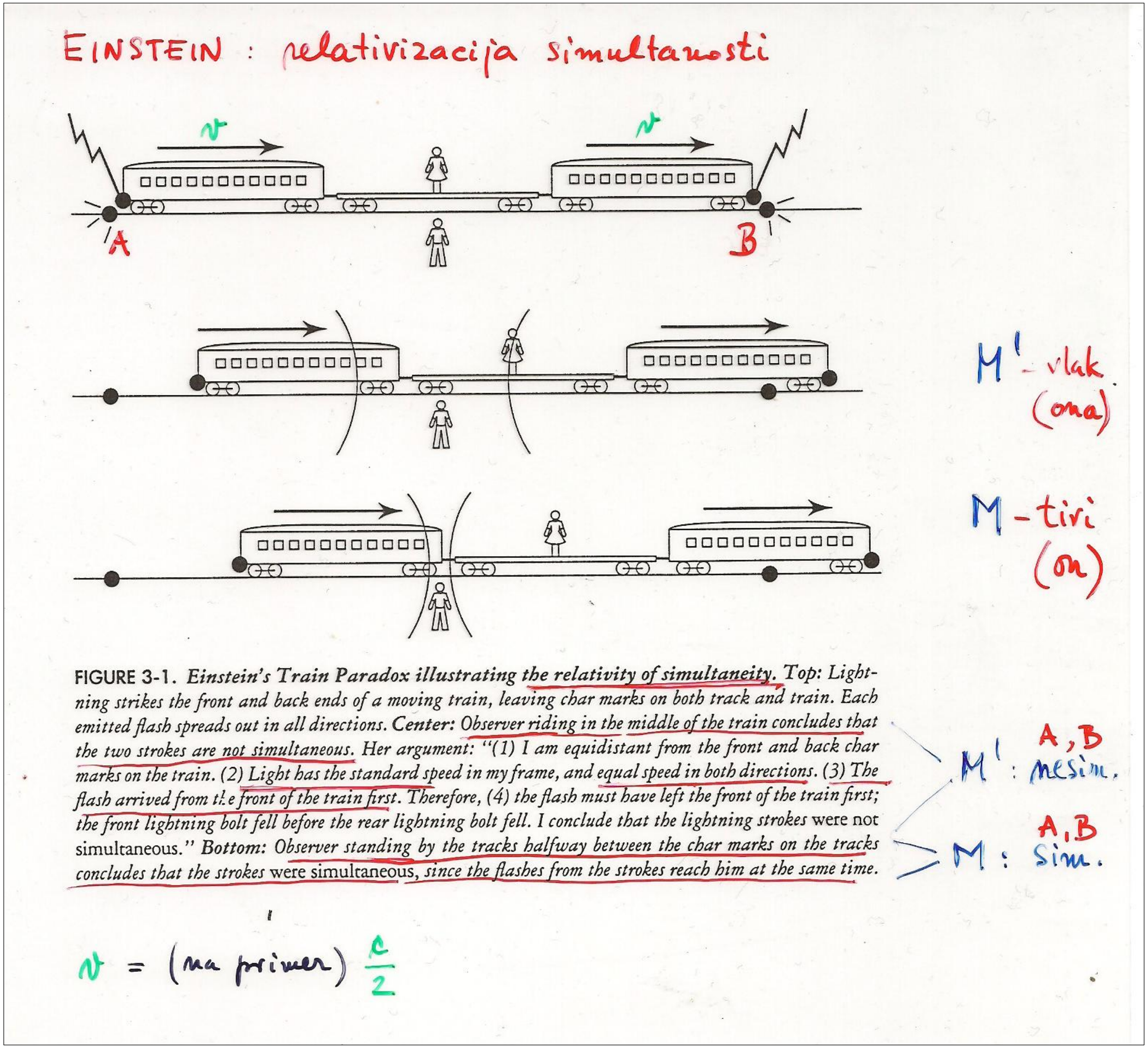
Razlaga raztezanja (dilatacije) časa
v SRT z "idealno svetlobno uro".
Nota bene: hitrost svetlobe *c* je
konstantna, vselej ista!

Izračun "relativističnega faktorja"
γ (gama), tu razmerja med *t* in *t*',
je računsko sorazmerno preprost:
dobimo ga s Pitagorovim izrekom.

Faktor γ je že pred Einsteinom določil
Hendrik Lorentz, po njem imenujemo
prehode med relativističnimi koordinatnimi sistemi (ali referenčnimi okviri)
v SRT Lorentzove transformacije.

Iz knjige: Abell, Morrison, Wolff,
Realm of the Universe
Sauders College Publishing, NY, 1994.

Einsteinova razlaga relativizacije simultanosti (dveh dogodkov A in B)



EINSTEIN : relativizacija simultanosti

FIGURE 3-1. *Einstein's Train Paradox illustrating the relativity of simultaneity. Top: Lightning strikes the front and back ends of a moving train, leaving char marks on both track and train. Each emitted flash spreads out in all directions.* **Center:** *Observer riding in the middle of the train concludes that the two strokes are not simultaneous. Her argument: "(1) I am equidistant from the front and back char marks on the train. (2) Light has the standard speed in my frame, and equal speed in both directions. (3) The flash arrived from the front of the train first. Therefore, (4) the flash must have left the front of the train first; the front lightning bolt fell before the rear lightning bolt fell. I conclude that the lightning strokes* were not simultaneous." ***Bottom:*** *Observer standing by the tracks halfway between the char marks on the tracks concludes that the strokes* were simultaneous, *since the flashes from the strokes reach him at the same time.*

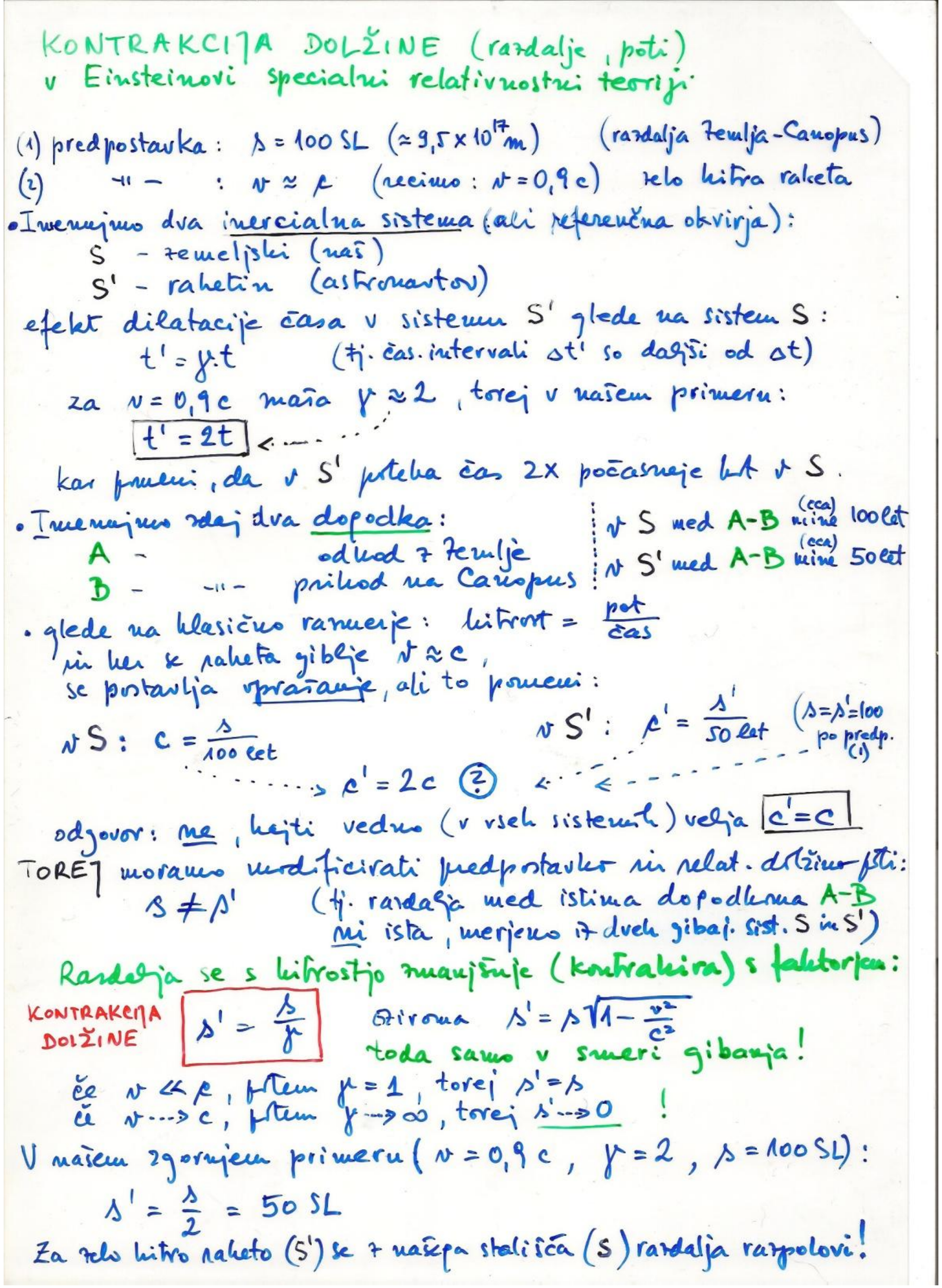
M' : A, B nesim.

M : A, B sim.

v = (na primer) $\frac{c}{2}$

Dogodka A in B,
ki sta simultana
(istočasna) za dečka
ob progi, nista
simultana za deklico
na hitrem vlaku –
zaradi konstantne
svetlobne hitrosti *c*.

Iz knjige (učbenika):
Taylor & Wheeler,
Spacetime Physics
(1992)



KONTRAKCIJA DOLŽINE (razdalje, poti) v Einsteinovi specialni relativnostni teoriji

(1) predpostavka: $s = 100$ SL ($\approx 9{,}5 \times 10^{17}$ m) (razdalja Zemlja-Canopus)

(2) -"- : $v \approx c$ (recimo: $v = 0{,}9c$) zelo hitra raketa

- Imenujmo dva <u>inercialna sistema</u> (ali referenčna okvirja):
  - S - zemeljski (naš)
  - S' - raketin (astronavtov)

efekt dilatacije časa v sistemu S' glede na sistem S:

$t' = \gamma \cdot t$ (tj. čas. intervali $\Delta t'$ so daljši od Δt)

za $v = 0{,}9c$ znaša $\gamma \approx 2$, torej v našem primeru:

$\boxed{t' = 2t}$

kar pomeni, da v S' poteka čas 2X počasneje kot v S.

- Imenujmo zdaj dva <u>dogodka</u>:
  - A - odhod z Zemlje
  - B - -"- prihod na Canopus

v S med A-B (cca) mine 100 let

v S' med A-B (cca) mine 50 let

- glede na klasično razmerje: hitrost = $\frac{\text{pot}}{\text{čas}}$
  in ker se raketa giblje $v \approx c$,
  se postavlja <u>vprašanje</u>, ali to pomeni:

v S: $c = \frac{s}{100 \text{ let}}$

v S': $c' = \frac{s'}{50 \text{ let}}$ ($s = s' = 100$ po predp. (1))

$c' = 2c$?

odgovor: <u>ne</u>, kajti vedno (v vseh sistemih) velja $\boxed{c' = c}$

TOREJ moramo modificirati predpostavko in relat. dolžino poti:

$s \neq s'$ (tj. razdalja med istima dogodkoma A-B <u>ni</u> ista, merjeno iz dveh gibaj. sist. S in S')

Razdalja se s hitrostjo zmanjšuje (kontrahira) s faktorjem:

KONTRAKCIJA DOLŽINE $\boxed{s' = \frac{s}{\gamma}}$ oziroma $s' = s\sqrt{1 - \frac{v^2}{c^2}}$

toda samo v smeri gibanja!

če $v \ll c$, potem $\gamma = 1$, torej $s' = s$

če $v \dashrightarrow c$, potem $\gamma \dashrightarrow \infty$, torej $s' \dashrightarrow 0$!

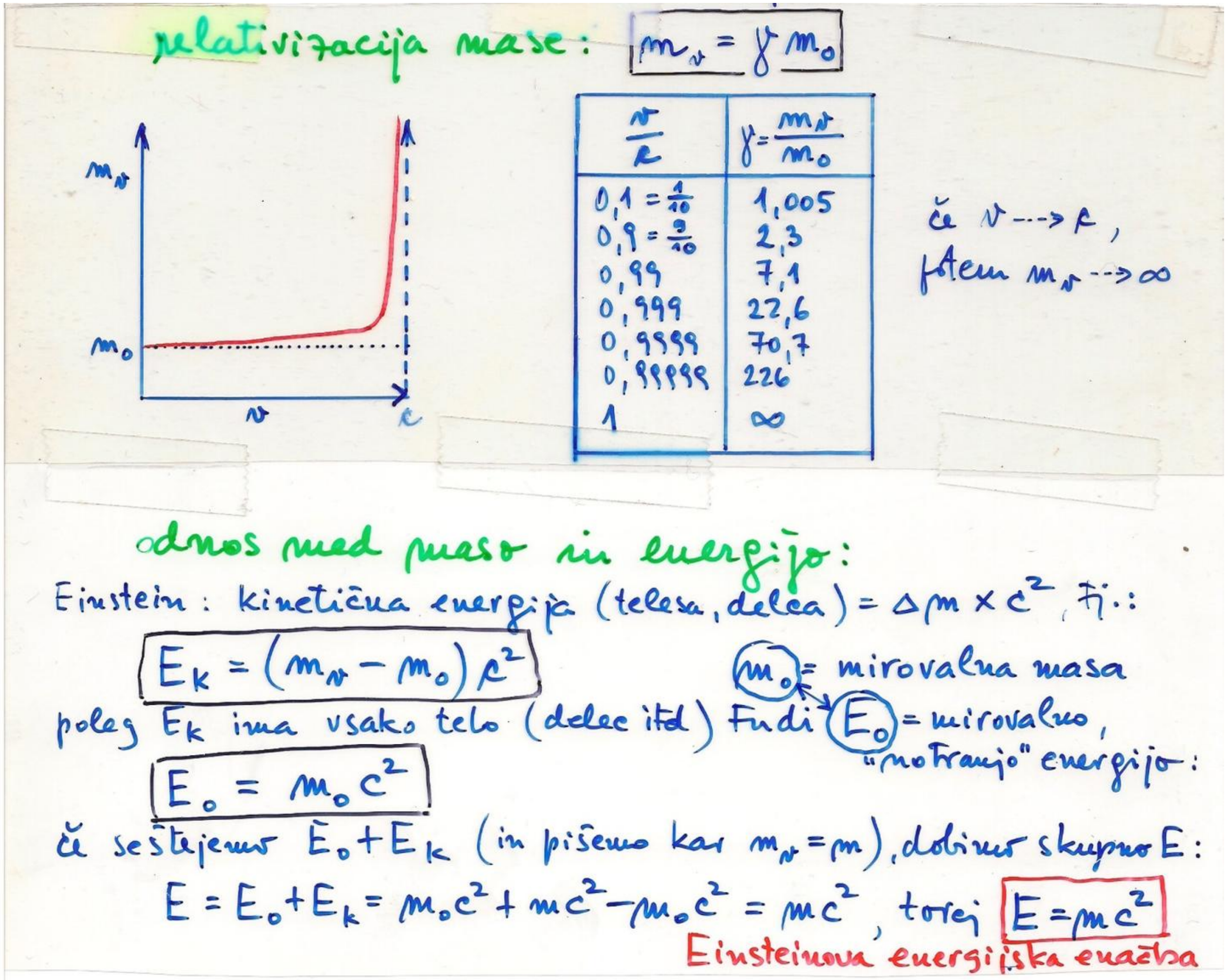
V našem zgornjem primeru ($v = 0{,}9c$, $\gamma = 2$, $s = 100$ SL):

$s' = \frac{s}{2} = 50$ SL

Za zelo hitro raketo (S') se z našega stališča (S) razdalja razpolovi!

Neformalna razlaga relativističnega skrčenja (kontrakcije) dolžine v SRT, na primeru potovanja z zelo hitro raketo na zvezdo Canopus, ki je od nas oddaljena približno 100 SL (svetlobnih let).

Neformalna razlaga relativizacije mase in izpeljave enačbe $E = mc^2$.



relativizacija mase: $\boxed{m_v = \gamma m_0}$

$\frac{v}{c}$	$\gamma = \frac{m_v}{m_0}$
$0{,}1 = \frac{1}{10}$	1,005
$0{,}9 = \frac{9}{10}$	2,3
0,99	7,1
0,999	22,6
0,9999	70,7
0,99999	226
1	∞

če $v \dashrightarrow c$,
potem $m_v \dashrightarrow \infty$

odnos med maso in energijo:

Einstein: kinetična energija (telesa, delca) = $\Delta m \times c^2$, tj.:

$$\boxed{E_k = (m_v - m_0)c^2}$$

m_0 = mirovalna masa

poleg E_k ima vsako telo (delec itd) tudi E_0 = mirovalno, "notranjo" energijo:

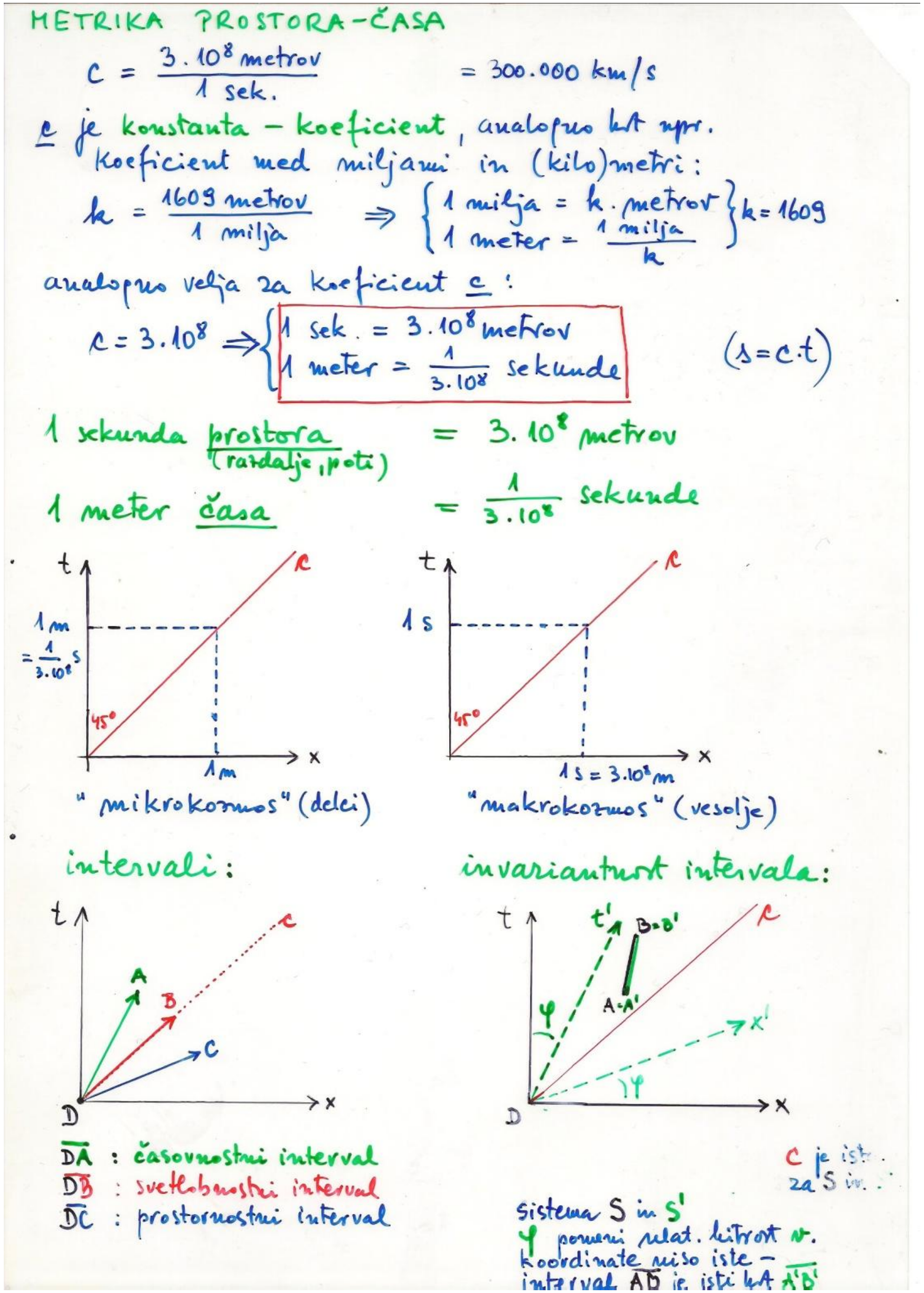
$$\boxed{E_0 = m_0 c^2}$$

če seštejemo $E_0 + E_k$ (in pišemo kar $m_v = m$), dobimo skupno E:

$$E = E_0 + E_k = m_0 c^2 + mc^2 - m_0 c^2 = mc^2, \text{ torej } \boxed{E = mc^2}$$

Einsteinova energijska enačba

Telo z mirovalno maso večjo od nič bi imelo pri svetlovni hitrosti *c* neskončno maso.



METRIKA PROSTORA-ČASA

$$c = \frac{3.10^8 \text{ metrov}}{1 \text{ sek.}} = 300.000 \text{ km/s}$$

c je konstanta – koeficient, analogno kot npr. koeficient med miljami in (kilo)metri:

$$k = \frac{1609 \text{ metrov}}{1 \text{ milja}} \Rightarrow \left\{ \begin{array}{l} 1 \text{ milja} = k.\text{metrov} \\ 1 \text{ meter} = \frac{1 \text{ milja}}{k} \end{array} \right\} k = 1609$$

analogno velja za koeficient c:

$$c = 3.10^8 \Rightarrow \left\{ \boxed{\begin{array}{l} 1 \text{ sek.} = 3.10^8 \text{ metrov} \\ 1 \text{ meter} = \frac{1}{3.10^8} \text{ sekunde} \end{array}} \right. \qquad (s = c.t)$$

1 sekunda prostora (razdalje, poti) $= 3.10^8$ metrov

1 meter časa $= \frac{1}{3.10^8}$ sekunde

"mikrokozmos" (delci)

"makrokozmos" (vesolje)

intervali:

$\overline{DA}$: časovnostni interval
$\overline{DB}$: svetlobnostni interval
$\overline{DC}$: prostornostni interval

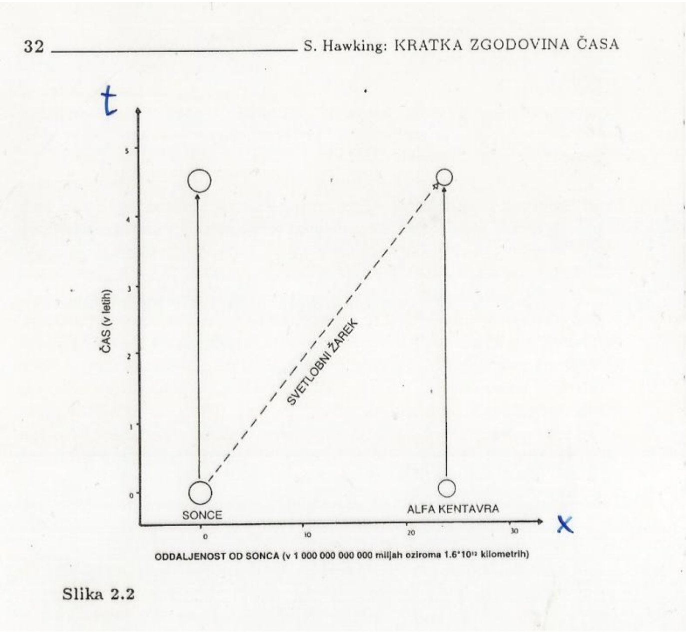
invariantnost intervala:

c je ist…
za S in…

sistema S in S'
φ pomeni relat. hitrost v.
koordinate niso iste –
interval $\overline{AB}$ je isti kot $\overline{A'B'}$

Einstein-Minkowskijeva metrika štirirazsežnega (4D) prostora-časa.

"Svetlobni stožci" (iz knjige: Stephen Hawking, *Kratka zgodovina časa*)



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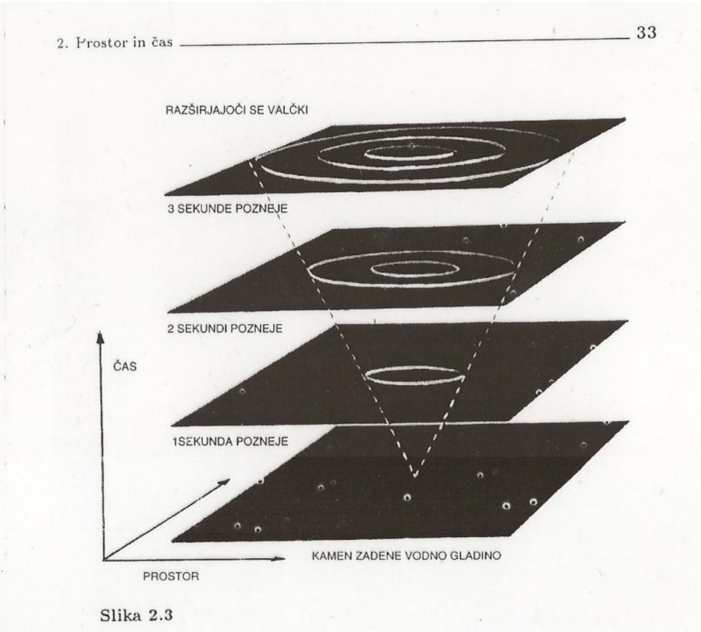
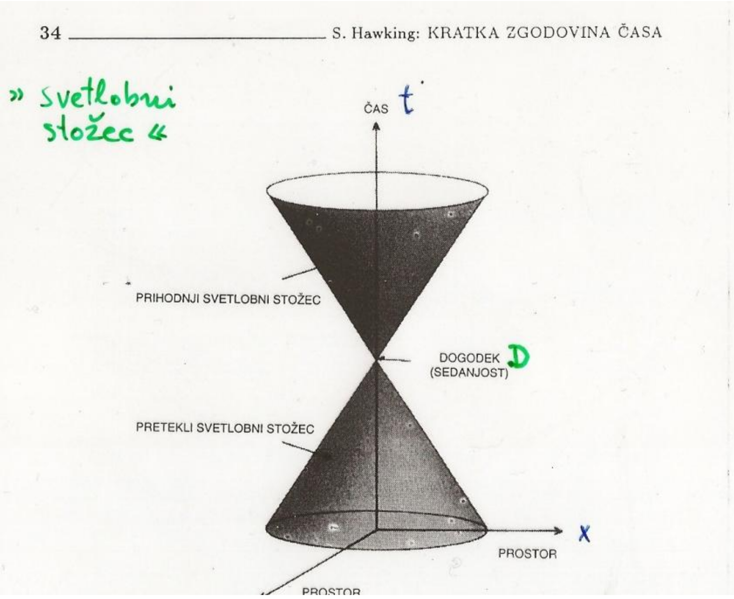
Slika 2.2



2. Prostor in čas 33

Slika 2.3



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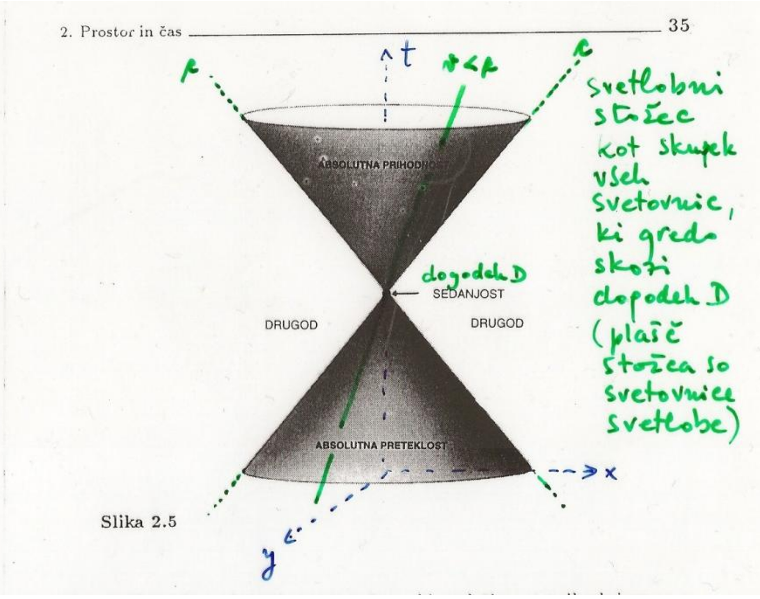
» svetlobni stožec «

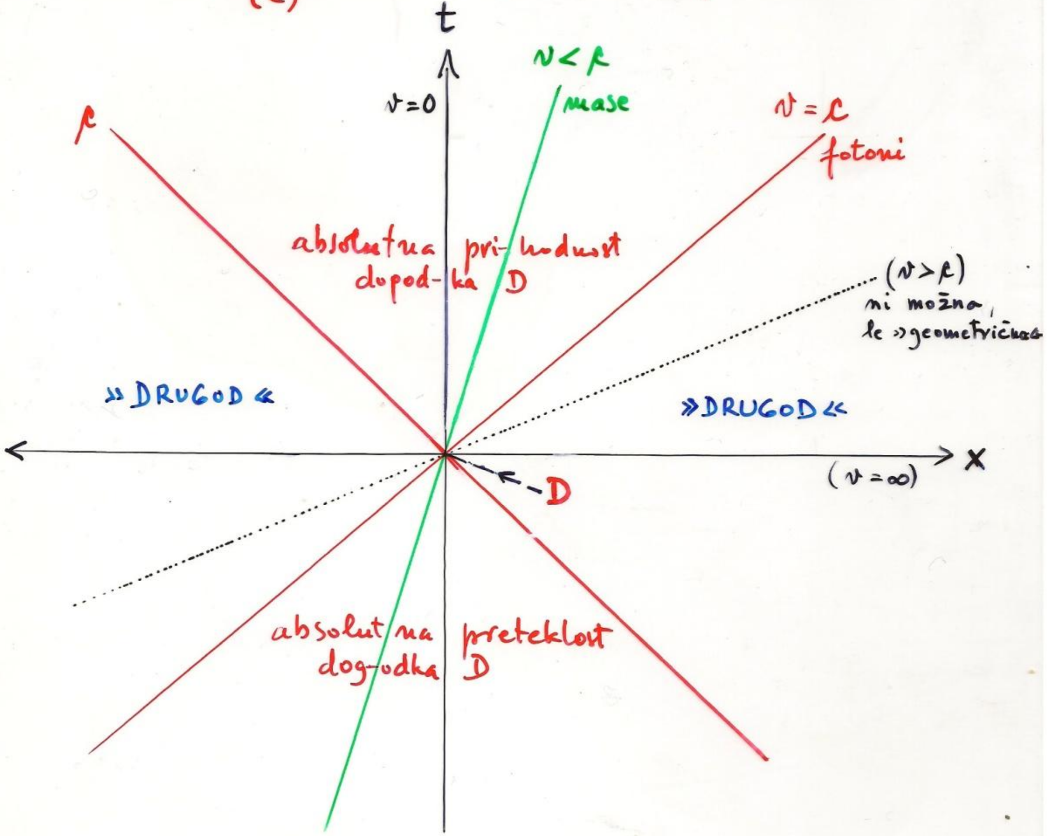


2. Prostor in čas 35

svetlobni stožec kot skupek vseh svetovnic, ki gredo skozi dogodek D (plašč stožca so svetovnice svetlobe)

Slika 2.5

Svetlobni »stožec«, shematično v 2 dimenzijah:
rdeči premici (c) sta meji svetlobnega »stožca«



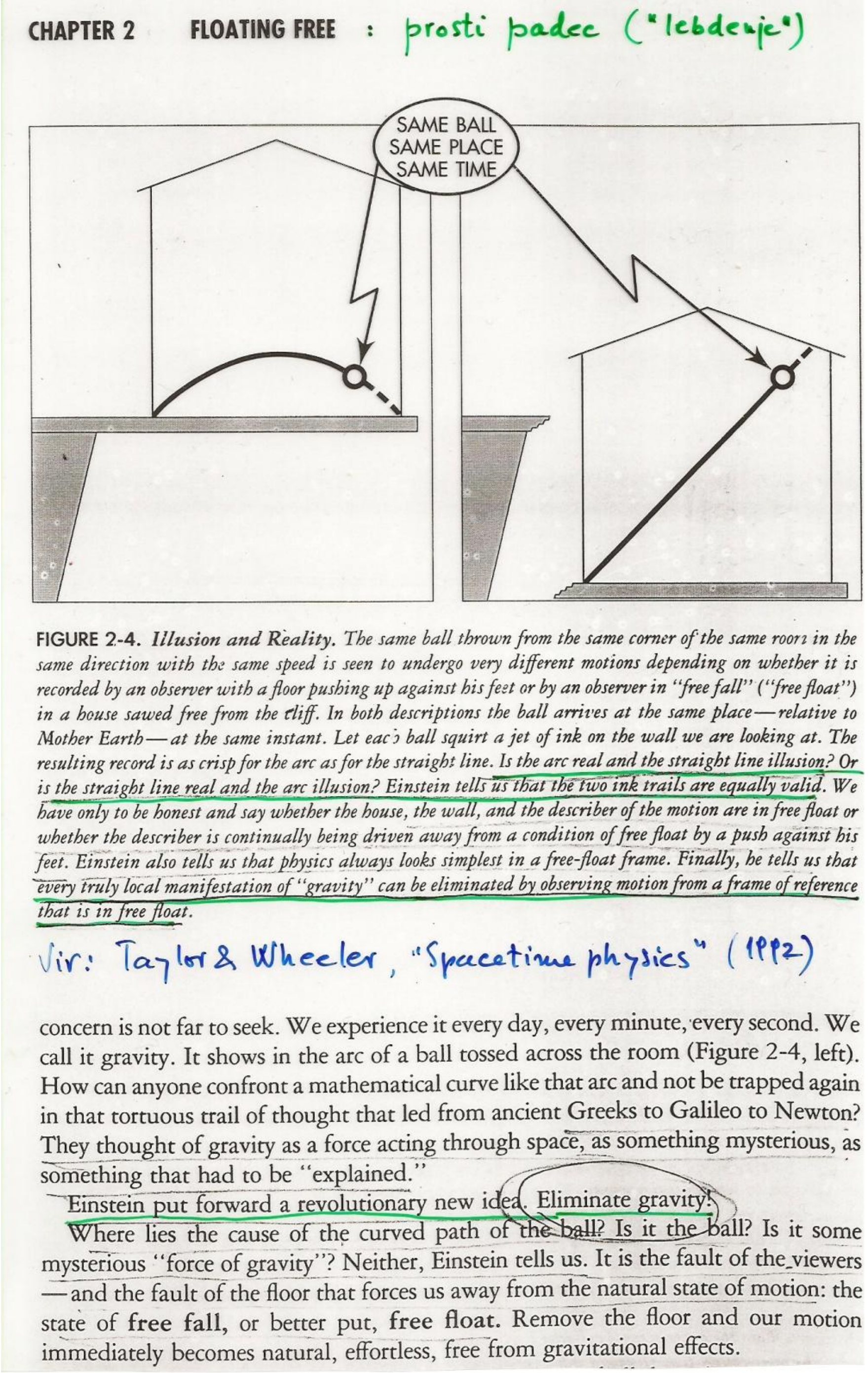
»DRUGOD« je vse, kar je izven »absolutne prihodnosti« in »absolutne preteklosti« — relativno, glede na dogodek D

- prihodnji svetlobni stožec dogodka D je množica vseh možnih učinkov tega dogodka
- pretekli svetlobni stožec dogodka D je množica vseh možnih vzrokov tega dogodka

povezava relativistične fizike z VZROČNOSTJO

Razlaga odnosov med “svetlobnimi stožci” (v 2D) in vzročnimi nizi

Einsteinovo “načelo ekvivalence” (1911)
kot temelj splošne teorije relativnosti (GRT, 1916):
“eliminacija gravitacije”, tj. izenačenje gravitacijske in inercialne mase oz. redukcija težnosti na pospešene referenčne okvire



CHAPTER 2 FLOATING FREE : prosti padec ("lebdenje")

FIGURE 2-4. *Illusion and Reality. The same ball thrown from the same corner of the same room in the same direction with the same speed is seen to undergo very different motions depending on whether it is recorded by an observer with a floor pushing up against his feet or by an observer in "free fall" ("free float") in a house sawed free from the cliff. In both descriptions the ball arrives at the same place—relative to Mother Earth—at the same instant. Let each ball squirt a jet of ink on the wall we are looking at. The resulting record is as crisp for the arc as for the straight line. Is the arc real and the straight line illusion? Or is the straight line real and the arc illusion? Einstein tells us that the two ink trails are equally valid. We have only to be honest and say whether the house, the wall, and the describer of the motion are in free float or whether the describer is continually being driven away from a condition of free float by a push against his feet. Einstein also tells us that physics always looks simplest in a free-float frame. Finally, he tells us that every truly local manifestation of "gravity" can be eliminated by observing motion from a frame of reference that is in free float.*

Vir: Taylor & Wheeler, "Spacetime physics" (1992)

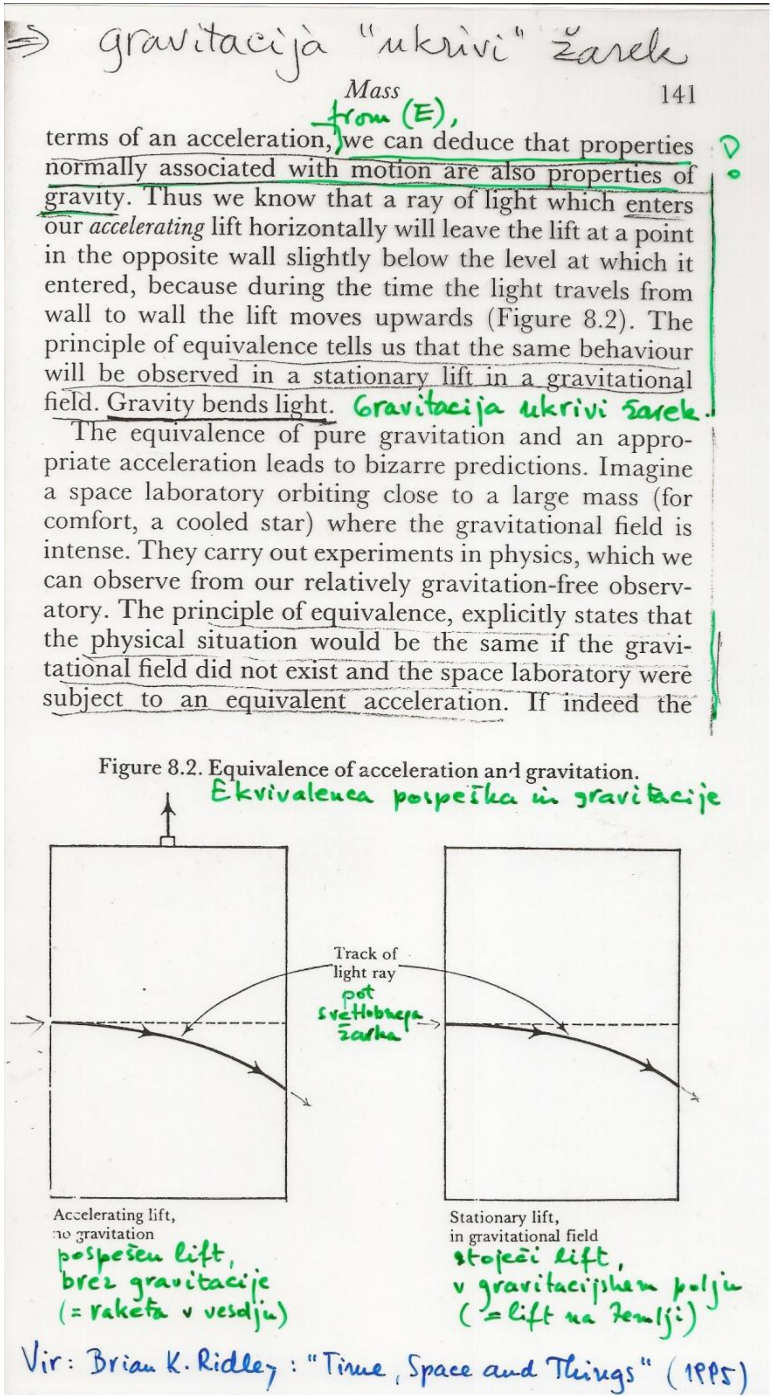
concern is not far to seek. We experience it every day, every minute, every second. We call it gravity. It shows in the arc of a ball tossed across the room (Figure 2-4, left). How can anyone confront a mathematical curve like that arc and not be trapped again in that tortuous trail of thought that led from ancient Greeks to Galileo to Newton? They thought of gravity as a force acting through space, as something mysterious, as something that had to be "explained."

Einstein put forward a revolutionary new idea. Eliminate gravity!

Where lies the cause of the curved path of the ball? Is it the ball? Is it some mysterious "force of gravity"? Neither, Einstein tells us. It is the fault of the viewers —and the fault of the floor that forces us away from the natural state of motion: the state of **free fall**, or better put, **free float**. Remove the floor and our motion immediately becomes natural, effortless, free from gravitational effects.

Iz knjige: Taylor & Wheeler,
Spacetime Physics (izd. 1992)

“Načelo ekvivalence”, primerjava lifta v gravitacijskem polju in rakete, ki pospešuje v breztežnostnem prostoru



⇒ gravitacija "ukrivi" žarek

Mass 141

terms of an acceleration, from (E), we can deduce that properties normally associated with motion are also properties of gravity. Thus we know that a ray of light which enters our *accelerating* lift horizontally will leave the lift at a point in the opposite wall slightly below the level at which it entered, because during the time the light travels from wall to wall the lift moves upwards (Figure 8.2). The principle of equivalence tells us that the same behaviour will be observed in a stationary lift in a gravitational field. Gravity bends light. Gravitacija ukrivi žarek. D

The equivalence of pure gravitation and an appropriate acceleration leads to bizarre predictions. Imagine a space laboratory orbiting close to a large mass (for comfort, a cooled star) where the gravitational field is intense. They carry out experiments in physics, which we can observe from our relatively gravitation-free observatory. The principle of equivalence, explicitly states that the physical situation would be the same if the gravitational field did not exist and the space laboratory were subject to an equivalent acceleration. If indeed the

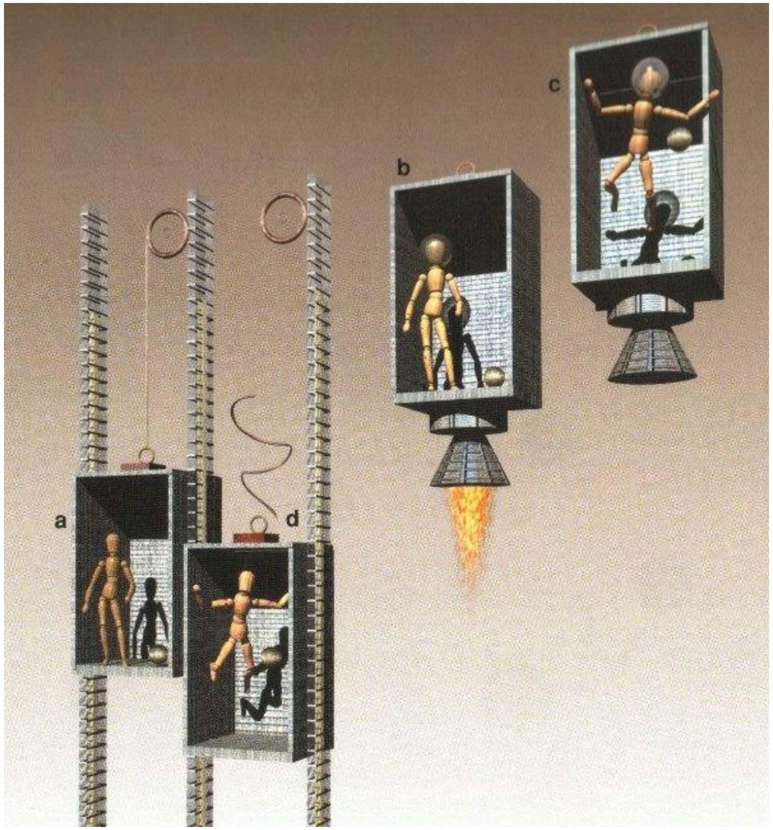
Figure 8.2. Equivalence of acceleration and gravitation.
Ekvivalenca pospeška in gravitacije

Track of light ray
pot svetlobnega žarka

Accelerating lift,
no gravitation
pospešen lift,
brez gravitacije
(= raketa v vesolju)

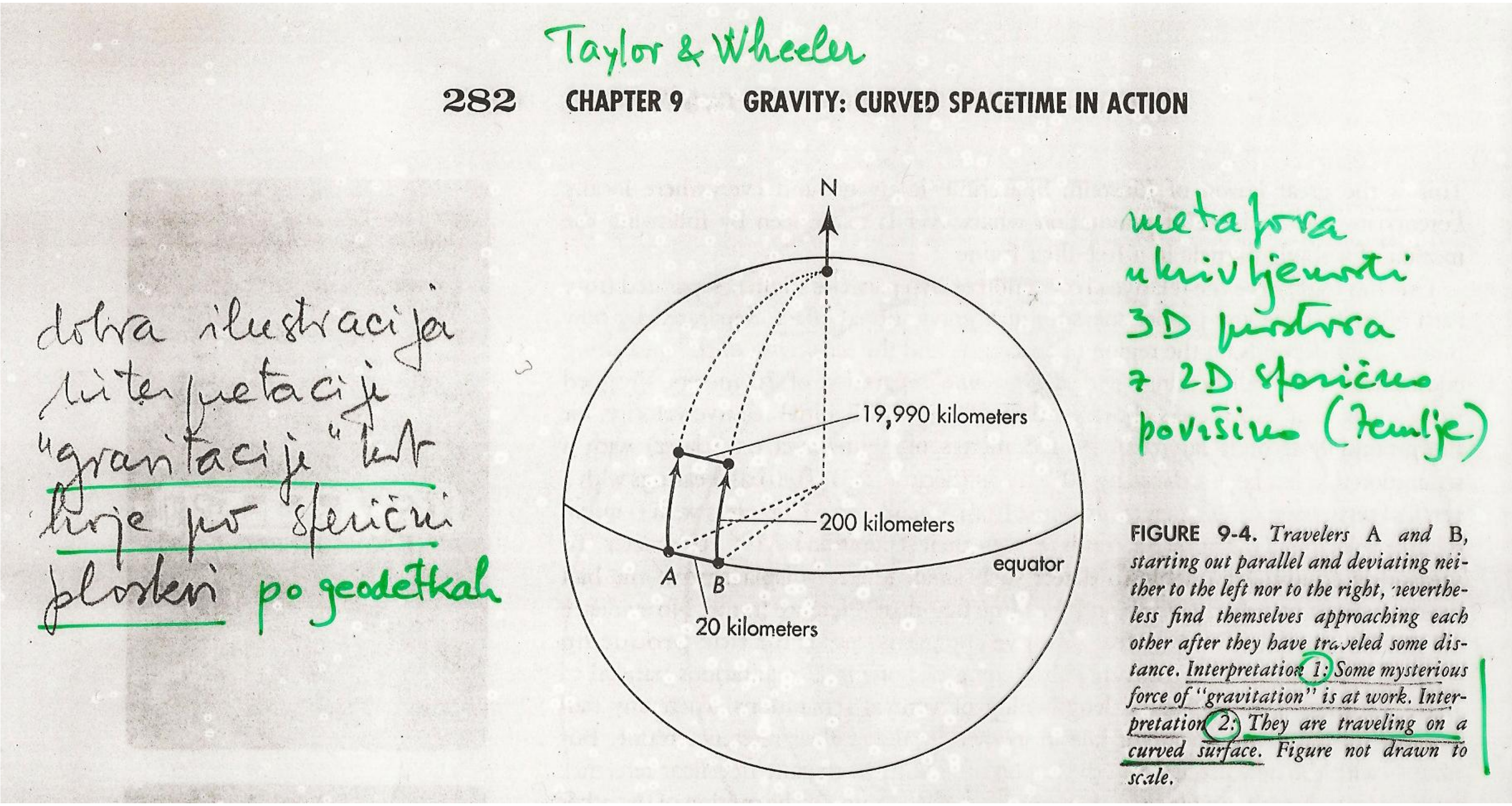
Stationary lift,
in gravitational field
stoječi lift,
v gravitacijskem polju
(= lift na Zemlji)

Vir: Brian K. Ridley: "Time, Space and Things" (1995)



Ilustracija iz knjige Stephena Hawkinga *Vesolje v orehovi lupini* (2001).
Prehod od (a) k (d) je fizično enak kot prehod od (b) k (c).

Gravitacija kot ukrivljenost prostora



Taylor & Wheeler

282 CHAPTER 9 GRAVITY: CURVED SPACETIME IN ACTION

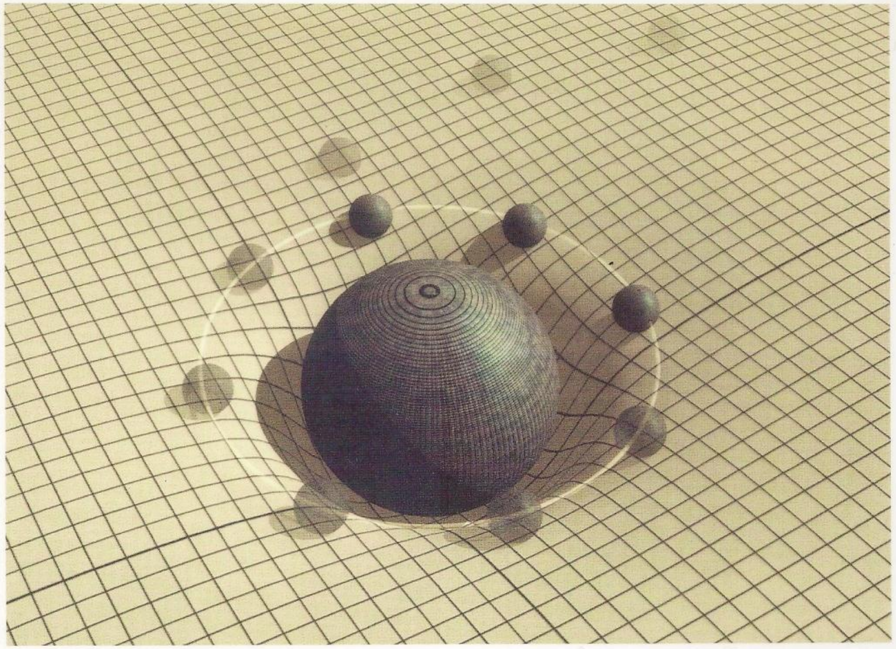
dobra ilustracija
interpretacije
"gravitacije" kot
linije po sferični
ploskvi po geodetkah

metafora
ukrivljenosti
3D prostora
z 2D sferično
površino (Zemlje)

FIGURE 9-4. *Travelers A and B, starting out parallel and deviating neither to the left nor to the right, nevertheless find themselves approaching each other after they have traveled some distance. Interpretation 1: Some mysterious force of "gravitation" is at work. Interpretation 2: They are traveling on a curved surface. Figure not drawn to scale.*

Geodetka je *najkrajša* razdalja med dvema "dogodkoma" (A in B) v poljubno ukrivljenem *prostoru* (tj. posplošitev premice iz evklidskega na neevklidske prostore) in obenem *najdaljša* razdalja med A in B v *času*.

Poti svetlobnih žarkov so "ničelne geodetke".



Einsteinova razlaga gibanja planeta okrog Sonca: planeta ne drži v orbiti neka “misteriozna sila”, ki naj bi delovala skozi prazen prostor (kot pri Newtonu), ampak planet “prosto pada” (tj., sledi svoji geodetki) v ukrivljenem prostoru-času, ki ga ukrivljajo same mase in/ali energije (v tem primeru Sonce).

Ilustracija iz knjige Stephena Hawkinga *Vesolje v orehovi lupini* (2001).

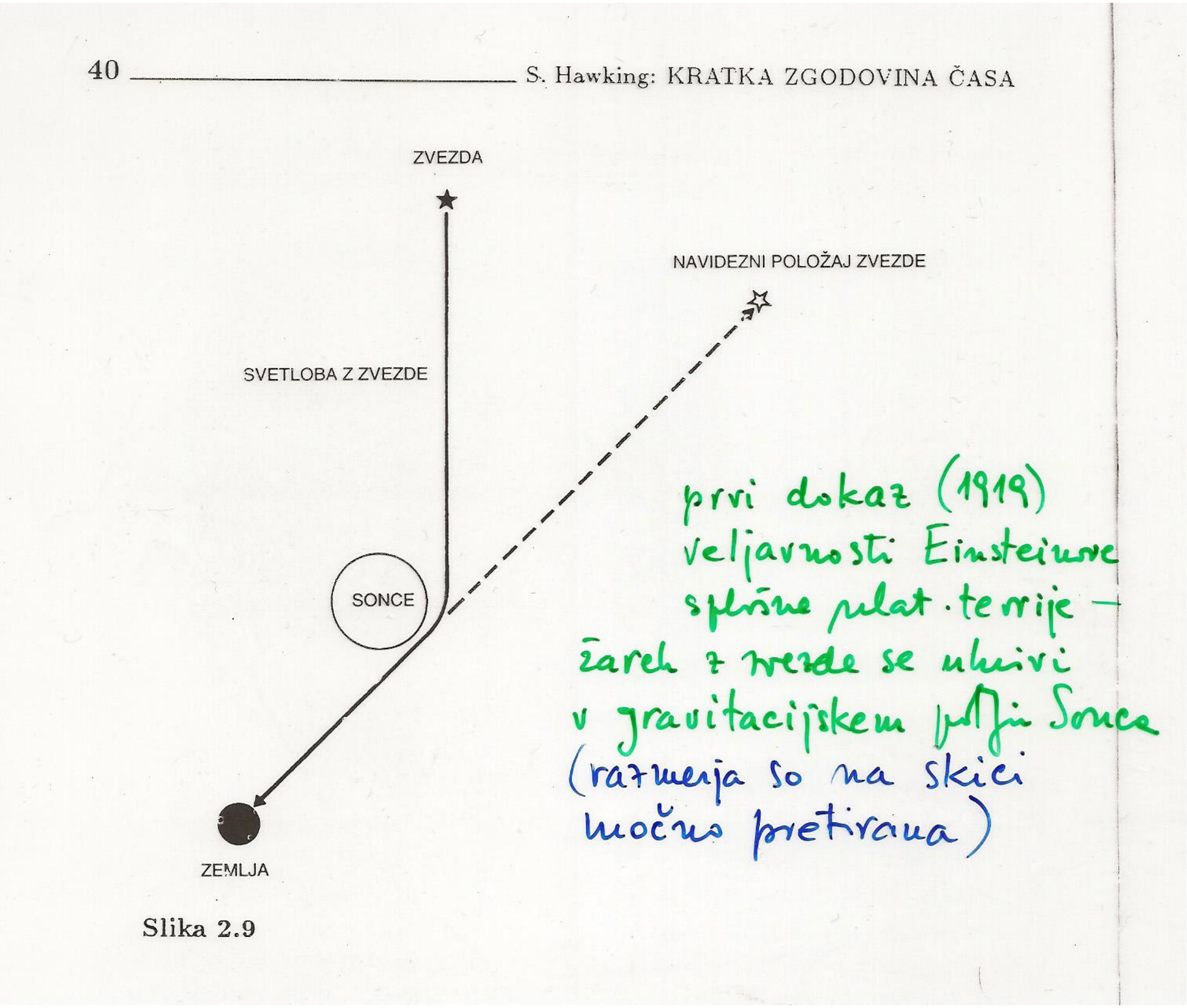
Einsteinova razlaga gravitacije –
lažje telo (recimo, planet) se giblje v ukrivljenem
prostoru okrog težjega (sonca).
(iz Hawkinga: "Vesolje v orehovi lupini")

John A. Wheeler:
"Prostor-čas pove snovi (tj. masi/energiji),
kako naj se giblje,
snov pove prostoru-času,
kako naj se ukrivi."

→ Einsteinove "enačbe polja" (1915):
splošna oblika teh slavnih 8 enačb je:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \text{oz.:} \quad G = k . T$$

Prvi dokaz veljavnosti Einsteinove splošne teorije relativnosti (1919)



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Slika 2.9

prvi dokaz (1919) veljavnosti Einsteinove splošne relat. teorije — žarek z zvezde se ukrivi v gravitacijskem polju Sonca

(razmerja so na skici močno pretirana)